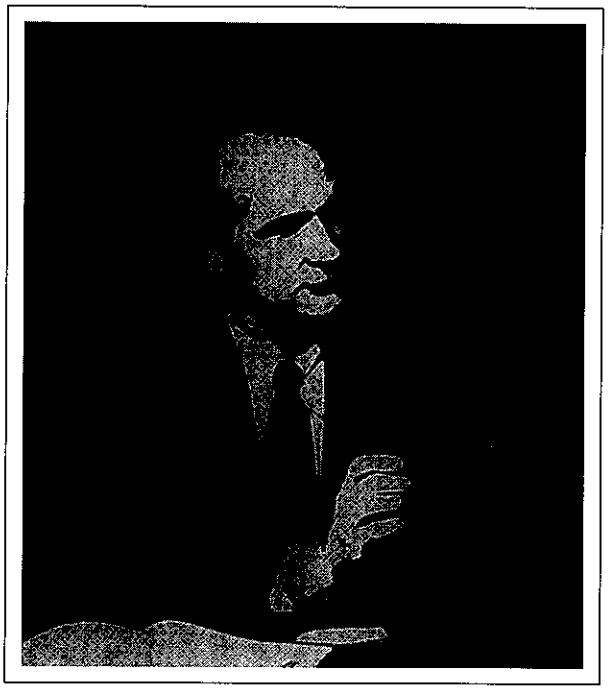
11 Investigations, Practical Problems, Puzzles



William Shockley Every time you use a calculator you are making use of integrated circuits which were developed from the first transistor. The transistor was invented by William Shockley, working with two scientists, in 1947. The three men shared the 1956 Nobel Prize for physics. The story of the invention is a good example of how mathematics can be used to solve practical problems.

The first electronic computers did not make use of transistors or integrated circuits and they were so big that they occupied whole rooms themselves. A modern computer which can carry out just the same functions can be carried around in a brief case.

11.1 Investigations

There are a large number of possible starting points for investigations here so it may be possible to allow students to choose investigations which appeal to them. On other occasions the same investigation may be set to a whole class.

Here are a few guidelines for you:

- If the set problem is too complicated try an easier case.
- Draw your own diagrams.
- Make tables of your results and be systematic.
- Look for patterns.
- Is there a rule or formula to describe the results?
- Can you *predict* further results?
- Can you *prove* any rules which you may find?

1. Opposite corners

Here the numbers are arranged in 10 columns.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

In the 2 × 2 square

$7 \times 18 = 126$
$8 \times 17 = 136$

7	8
17	18

the difference between them is 10.

In the 3 × 3 square

$12 \times 34 = 408$
$14 \times 32 = 448$

12	13	14
22	23	24
32	33	34

the difference between them is 40.

Investigate to see if you can find any rules or patterns connecting the size of square chosen and the difference.

If you find a rule, use it to *predict* the difference for larger squares.

Test your rule by looking at squares like 8 × 8 or 9 × 9.

Can you *generalise* the rule?
[What is the difference for a square of size $n \times n$?]

Can you prove the rule?
Hint:
In a 3 × 3 square ...

x		?
?		?

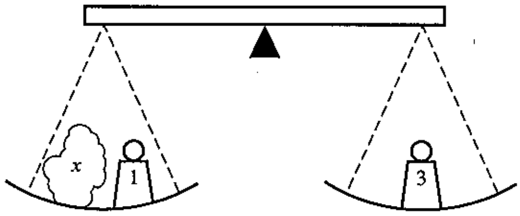
What happens if the numbers are arranged in six columns or seven columns?

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19					

1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22						

2. Weighing scales

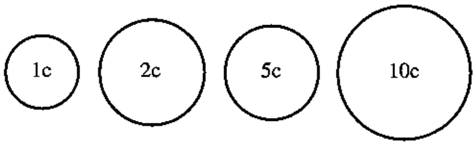
In the diagram we are measuring the weight of the package x using two weights.
If the scales are balanced, x must be 2 kg.



Show how you can measure all the weights from 1 kg to 10 kg using three weights: 1 kg, 3 kg, 6 kg.
It is possible to measure all the weights from 1 kg to 13 kg using a different set of three weights. What are the three weights?
It is possible to measure all the weights from 1 kg to 40 kg using four weights. What are the weights?

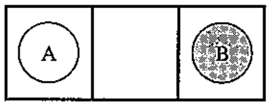
3. Buying stamps

You have one 1c, one 2c, one 5c and one 10c coin.
You can buy stamps of any value you like, but you must give the exact money.
How many different value stamps can you buy?
Suppose you now have one 1c, one 2c, one 5c, one 10c, one 20c, one 50c and one $1 note.
How many different value stamps can you buy now?



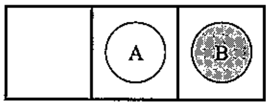
4. Frogs

This is a game invented by a French mathematician called Lucas.

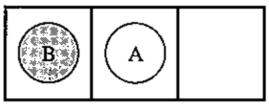


Aim: To swap the positions of the discs so that they end up the other way round (with a space in the middle).
Rules 1. A disc can slide one square in either direction onto an empty square.
2. A disc can hop over one adjacent disc of the other colour provided it can land on an empty square.

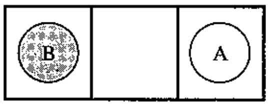
Example (a) Slide Ⓐ one square to the right.



(b) Ⓑ hops over Ⓐ to the left.

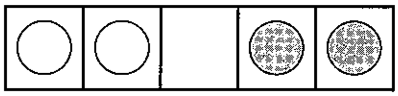


(c) Slide Ⓐ one square to the right.

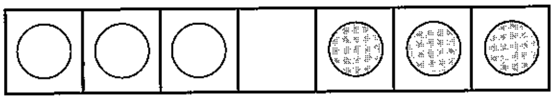


We took 3 moves.

1. Look at the diagram. What is the smallest number of moves needed for two discs of each colour?



2. Now try three discs of each colour. Can you complete the task in 15 moves?



3. Try four discs of each colour.
   Now look at your results and try to find a formula which gives the least number of moves needed for any number of discs x. It may help if you count the number of 'hops' and 'slides' separately.
4. Try the game with a different number of discs on each side. Say two reds and three blues. Play the game with different combinations and again try to find a formula giving the number of moves for x discs of one colour and y discs of another colour.

5. Triples

In this investigation a *triple* consists of three whole numbers in a definite order. For example, (4, 2, 1) is a triple and (1, 4, 2) is a different triple.

The three numbers in a triple do not have to be different. For example, (2, 2, 3) is a triple but (2, 0, 1) is not a triple because 0 is not allowed.

The *sum* of a triple is found by adding the three numbers together. So the sum of (4, 2, 1) is 7.

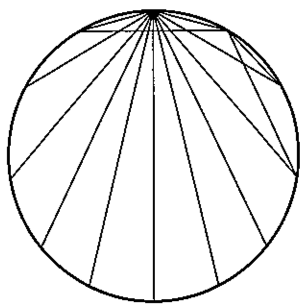
Investigate how many different triples there are with a given sum. See what happens to the number of different triples as the sum is changed.

If you find any pattern, try to explain why it occurs.

How many different triples are there whose sum is 22?

6. Mystic rose

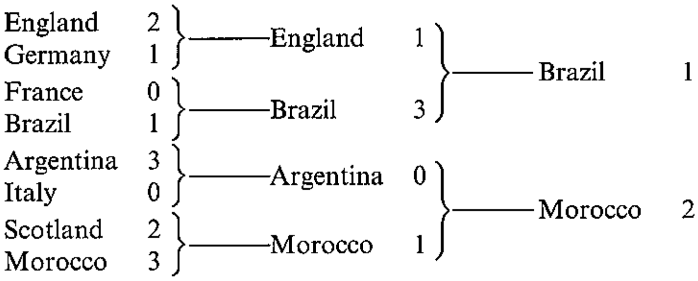
Straight lines are drawn between each of the 12 points on the circle. Every point is joined to every other point. How many straight lines are there?



Suppose we draw a mystic rose with 24 points on the circle. How many straight lines are there?
How many straight lines would there be with n points on the circle?

7. Knockout competition

Eight teams reach the 'knockout' stage of the World Cup.



How would you organise a knockout competition if there were 12 teams? Or 15?

How many matches are played up to and including the final if there are:

(a) 8 teams,
(b) 12 teams,
(c) 15 teams,
(d) 23 teams,
(e) n teams?

In a major tournament like Wimbledon, the better players are seeded from 1 to 16. Can you organise a tournament for 32 players so that, if they win all their games:

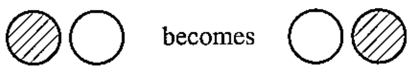
(a) seeds 1 and 2 can meet in the final,
(b) seeds 1, 2, 3 and 4 can meet in the semi-finals,
(c) seeds 1, 2, 3, 4, 5, 6, 7, 8 can meet in the quarter-finals?

8. Discs

(a) You have five black discs and five white discs which are arranged in a line as shown.

We want to get all the black discs to the right-hand end and all the white discs to the left-hand end.

The only move allowed is to interchange two neighbouring discs.



How many moves does it take?
How many moves would it take if we had fifty black discs and fifty white discs arranged alternately?

(b) Suppose the discs are arranged in pairs

... etc.

How many moves would it take if we had fifty black discs and fifty white discs arranged like this?
[Hint: In both cases work with a smaller number of discs until you can see a pattern.]

(c) Now suppose you have three colours black, white and green arranged alternately.

... etc.

You want to get all the black discs to the right, the green discs to the left and the white discs in the middle.
How many moves would it take if you have 30 discs of each colour?

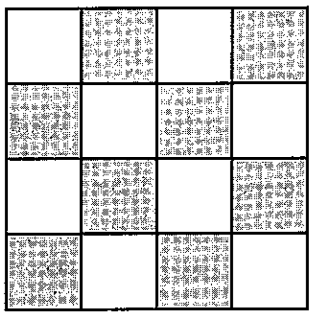
9. Chess board

Start with a small board, just 4×4.

How many squares are there? [It is not just 16!]

How many squares are there on an 8×8 chess board?

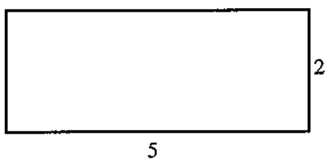
How many squares are there on an $n \times n$ chess board?



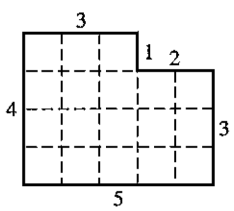
10. Area and perimeter

This is about finding different shapes in which the area is numerically equal to the perimeter.

This rectangle has an area of 10 square units and a perimeter of 14 units, so we will have to try another one.
There are some suggestions below but you can investigate shapes of your own choice if you prefer.

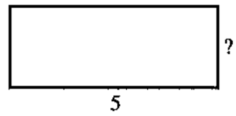


(a) Find rectangles with equal area and perimeter. After a while you can try adding on bits like this.



area = 18
perimeter = 18

(b) Suppose one dimension of the rectangle is fixed.
In this rectangle the length is 5 units.



(c) Try right-angled triangles and equilateral triangles.
(d) Try circles, semi-circles and so on.
(e) How about three-dimensional shapes? Now we are looking for cuboids, spheres, cylinders in which the volume is numerically equal to the surface area.
(f) Can you find any connection between the square with equal area and perimeter and the circle with equal area and perimeter? How about the equilateral triangle with equal area and perimeter?

11. Happy numbers (and more)

(a) Take the number 23.
Square the digits and add.

$$2\quad 3$$
$$2^2 + 3^2 = 1\quad 3$$
$$1^2 + 3^2 = 1\quad 0$$
$$1^2 + 0^2 = 1$$

The sequence ends at 1 and we call 23 a 'happy' number. Investigate for other numbers. Here are a few suggestions: 70, 85, 49, 44, 14, 15, 94.

(b) Now change the rule. Instead of squaring the digits we will cube them.

$$2\quad 1$$
$$2^3 + 1^3 = 0\quad 9$$
$$0^3 + 9^3 = 7\quad 2\quad 9$$
$$7^3 + 2^3 + 9^3 = 1\quad 0\quad 8\quad 0$$
$$1^3 + 0^3 + 8^3 = 5\quad 1\quad 3$$
$$5^3 + 1^3 + 3^3 = 153$$

And now we are stuck because 153 leads to 153 again. Investigate for numbers of your own choice. Do any numbers lead to 1?

12. Prime numbers

Write all the numbers from 1 to 104 in eight columns and draw a ring around the prime numbers 2, 3, 5 and 7.

1	(2)	(3)	4	(5)	6	(7)	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25							

If we cross out all the multiples of 2, 3, 5 and 7, we will be left with all the prime numbers below 104. Can you see why this works?

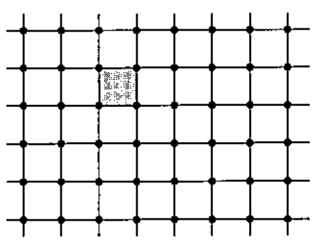
Draw *four* lines to eliminate the multiples of 2.
Draw *six* lines to eliminate the multiples of 3.
Draw *two* lines to eliminate the multiples of 7.
Cross out all the numbers ending in 5.

Put a ring around all the prime numbers less than 104.
[Check there are 27 numbers.]

Many prime numbers can be written as the sum of two squares. For example $5 = 2^2 + 1^2$, $13 = 3^2 + 2^2$. Find all the prime numbers in your table which can be written as the sum of two squares. Draw a red ring around them in the table.
What do you notice?
Check any 'gaps' you may have found.

Extend the table up to 200 and see if the pattern continues. In this case you will need to eliminate the multiples of 11 and 13 as well.

13. Squares



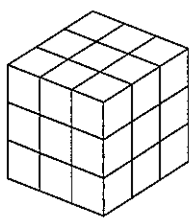
For this investigation you need either dotted paper or squared paper.

The shaded square has an area of 1 unit.
Can you draw a square, with its corners on the dots, with an area of 2 units?
Can you draw a square with an area of 3 units?
Can you draw a square with an area of 4 units?

Investigate for squares up to 100 units.

For which numbers x can you draw a square of area x units?

14. Painting cubes



The large cube below consists of 27 unit cubes.
All six faces of the large cube are painted green.

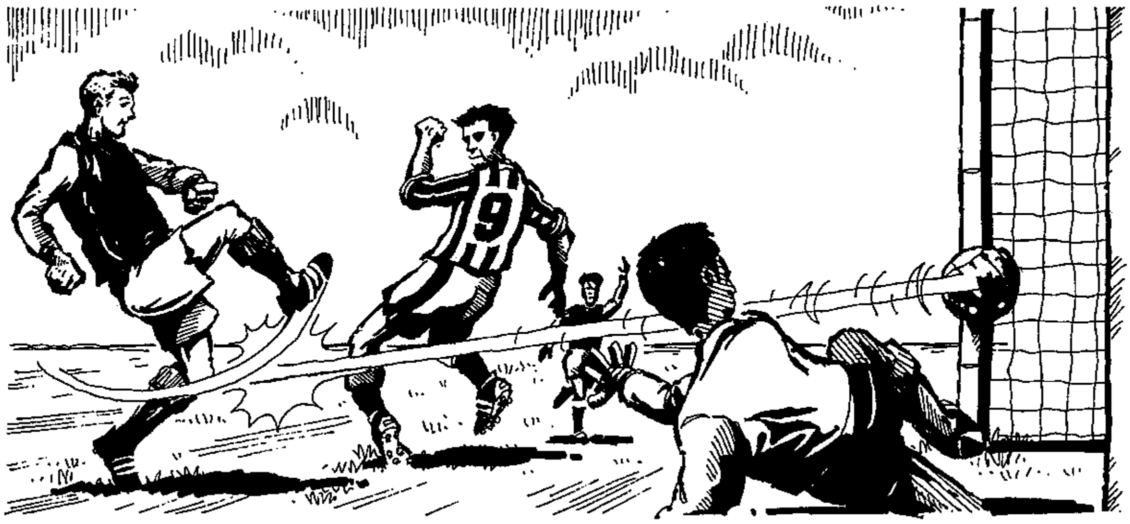
How many unit cubes have 3 green faces?
How many unit cubes have 2 green faces?
How many unit cubes have 1 green face?
How many unit cubes have 0 green faces?

Suppose the large cube is $20 \times 20 \times 20$.
Answer the four questions above.

Answer the four questions for the cube which is $n \times n \times n$.

15. Final score

The final score in a football match was 3–2. How many different scores were possible at half-time?



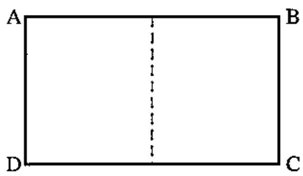
Investigate for other final scores where the difference between the teams is always one goal [1–0, 5–4 etc]. Is there a pattern or rule which would tell you the number of possible half-time scores in a game which finished 58–57?

Suppose the game ends in a draw. Find a rule which would tell you the number of possible half-time scores if the final score was 63–63.

Investigate for other final scores [3–0, 5–1, 4–2 etc].

16. Cutting paper

The rectangle ABCD is cut in half to give two smaller rectangles.



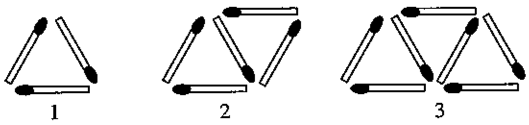
Each of the smaller rectangles is mathematically similar to the large rectangle. Find a rectangle which has this property.

What happens when the small rectangles are cut in half? Do they have the same property?

Why is this a useful shape for paper used in business?

17. Matchstick shapes

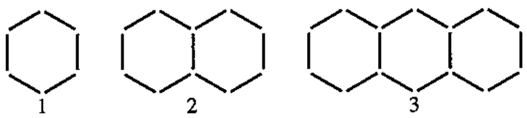
(a) Here we have a sequence of matchstick shapes



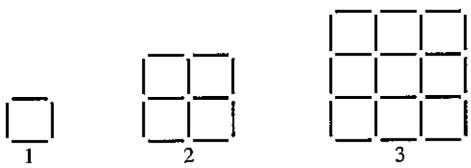
Can you work out the number of matches in the 10th member of the sequence? Or the 20th member of the sequence? How about the *n*th member of the sequence?

(b) Now try to answer the same questions for the patterns below. Or you may prefer to design patterns of your own.

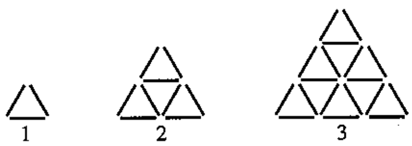
(i)



(ii)

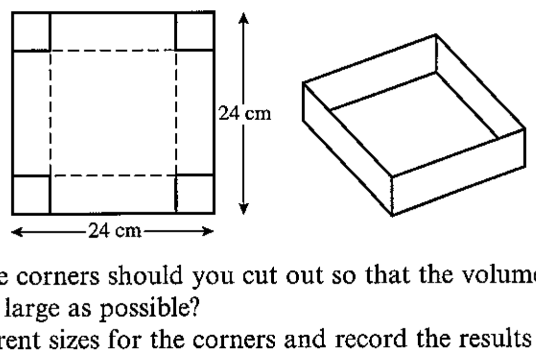


(iii)



18. Maximum box

(a) You have a square sheet of card 24 cm by 24 cm. You can make a box (without a lid) by cutting squares from the corners and folding up the sides.



What size corners should you cut out so that the volume of the box is as large as possible?
Try different sizes for the corners and record the results in a table:

length of the side of the corner square (cm)	dimensions of the open box (cm)	volume of the box (cm^3)
1	22 × 22 × 1	484
2		
–		
–		

Now consider boxes made from different sized cards:
15 cm × 15 cm and 20 cm by 20 cm.
What size corners should you cut out this time so that the volume of the box is as large as possible?

Is there a connection between the size of the corners cut out and the size of the square card?

(b) Investigate the situation when the card is not square. Take rectangular cards where the length is twice the width (20 × 10, 12 × 6, 18 × 9 etc).
Again, for the maximum volume is there a connection between the size of the corners cut out and the size of the original card?

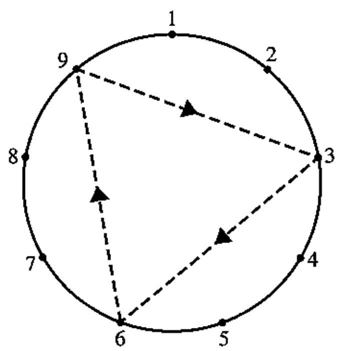
19. Digit sum

Take the number 134.
Add the digits $1 + 3 + 4 = 8$.
The digit sum of 134 is 8.

Take the number 238.
$2 + 3 + 8 = 13$ [We continue if the sum is more than 9].
$1 + 3 = 4$
The digit sum of 238 is 4.
Consider the multiples of 3:

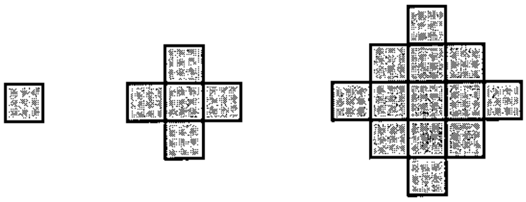
Number	3	6	9	12	15	18	21	24	27	30	33	36
Digit sum	3	6	9	3	6	9	3	6	9	3	6	9

The digit sum is always 3, 6,or 9.
These numbers can be shown on a circle.
Investigate the pattern of the digit sums for multiples of:
(a) 2 (b) 5 (c) 6 (d) 7 (e) 8
(f) 9 (g) 11 (h) 12 (i) 13
Is there any connection between numbers where the pattern of the digit sums is the same?
Can you (without doing all the usual working) predict what the pattern would be for multiples of 43? Or 62?



20. An expanding diagram

Look at the series of diagrams below.



1 2 3

Each time new squares are added all around the outside of the previous diagram.

Draw the next few diagrams in the series and count the number of squares in each one.

How many squares are there in diagram number 15 or in diagram number 50?

What happens if we work in three dimensions? Instead of adding squares we add cubes all around the outside. How many cubes are there in the fifth member of the series or the fifteenth?

21. Fibonacci sequence

Fibonacci was the nickname of the Italian mathematician Leonardo de Pisa (A.D. 1170–1250). The sequence which bears his name has fascinated mathematicians for hundreds of years. You can if you like join the Fibonacci Association which was formed in 1963.

Here is the start of the sequence
1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ...

There are no prizes for working out the next term!

The sequence has many interesting properties to investigate. Here are a few suggestions.

(a) Add three terms.
 $1 + 1 + 2$, $1 + 2 + 3$, etc.
 Add four terms.
(b) Add squares of terms
 $1^2 + 1^2, 1^2 + 2^2, 2^2 + 3^2, \ldots$
(c) Ratios
 $\frac{1}{1} = 1, \frac{2}{1} = 2, \frac{3}{2} = 1{\cdot}5, \ldots$
(d) In fours | 2 3 5 8 |
 $2 \times 8 = 16, 3 \times 5 = 15$
(e) In threes | 3 5 8 |
 $3 \times 8 = 24, 5^2 = 25$

(f) In sixes | 1 1 2 3 5 8 |

square and add the first five numbers

$1^2 + 1^2 + 2^2 + 3^2 + 5^2 = 40$

$5 \times 8 = 40$.

Now try seven numbers from the sequence, or eight ...

(g) Take a group of 10 consecutive terms. Compare the sum of the 10 terms with the seventh member of the group.

22. Alphabetical order

A teacher has four names on a piece of paper which are in no particular order (say Smith, Jones, Biggs, Eaton). He wants the names in alphabetical order.

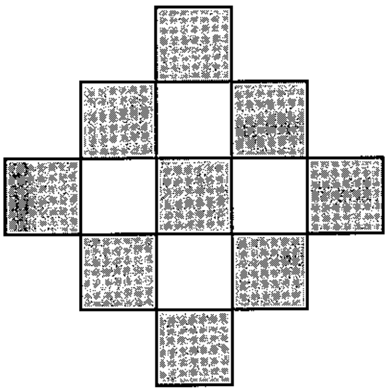
One way of doing this is to interchange each pair of names which are clearly out of order.
So he could start like this; S J B E
the order becomes J S B E
He would then interchange S and B.

Using this method, what is the largest number of interchanges he could possibly have to make?

What if he had thirty names, or fifty?

23. Mr Gibson's job

Mr Gibson's job is counting tiles of the black or white variety. When he is bored Mr Gibson counts the tiles by placing them in a pattern consisting of alternate black and white tiles. This one is five tiles across and altogether there are 13 tiles in the pattern.

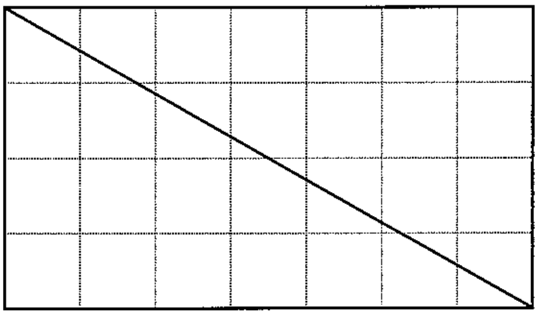


He makes the pattern so that there are always black tiles all around the outside. Draw the pattern which is nine tiles across. You should find that there are 41 tiles in the pattern.

How many tiles are there in the pattern which is 101 tiles across?

24. Diagonals

In a 4×7 rectangle the diagonal passes through 10 squares.



Draw rectangles of your own choice and count the number of squares through which the diagonal passes.

A rectangle is 640×250. How many squares will the diagonal pass through?

25. Biggest number

A calculator has the following buttons:

[+] [−] [×] [÷] [(] [)] [y^x] [=]

Also the only digits buttons which work are the '1', '2' and '3'.

(a) You can press any button, but only once.
What is the biggest number you can get?
(b) Now the '1', '2', '3' and '4' buttons are working.
What is the biggest number you can get?
(c) Investigate what happens as you increase the number of digits which you can use.

26. What shape tin?

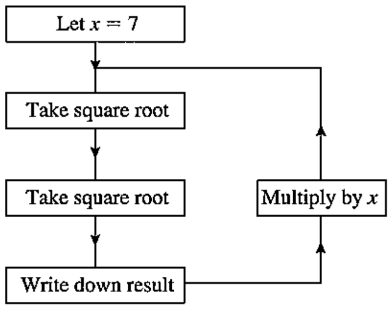
We need a cylindrical tin which will contain a volume of $600\,cm^3$ of drink.
What shape should we make the tin so that we use the minimum amount of metal?
In other words, for a volume of $600\,cm^3$, what is the smallest possible surface area?
Hint: Make a table.

r	h	A
2	?	?
3	?	?
⋮		

What shape tin should we design to contain a volume of $1000\,cm^3$?

27. Find the connection

Work through the flow diagram several times, using a calculator.



What do you notice?
Try different numbers for x (suggestions: 11, 5, 8, 27)
What do you notice?

What happens if you take the square root three times?

Suppose in the flow diagram you change
'Multiply by x' to 'Divide by x'. What happens now?

Suppose in the flow diagram you change
'Multiply by x' to 'Multiply by x^2'. What happens now?

28. Spotted shapes

For this investigation you need dotted paper. If you have not got any, you can make your own using a felt tip pen and squared paper.

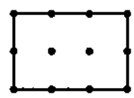


Diagram 1

The rectangle in Diagram 1 has 10 dots on the perimeter ($p = 10$) and 2 dots inside the shape ($i = 2$). The area of the shape is 6 square units ($A = 6$)

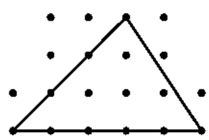


Diagram 2

The triangle in Diagram 2 has 9 dots on the perimeter ($p = 9$) and 4 dots inside the shape ($i = 4$). The area of the triangle is $7\frac{1}{2}$ square units ($A = 7\frac{1}{2}$)

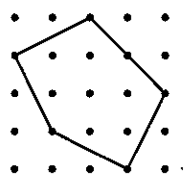
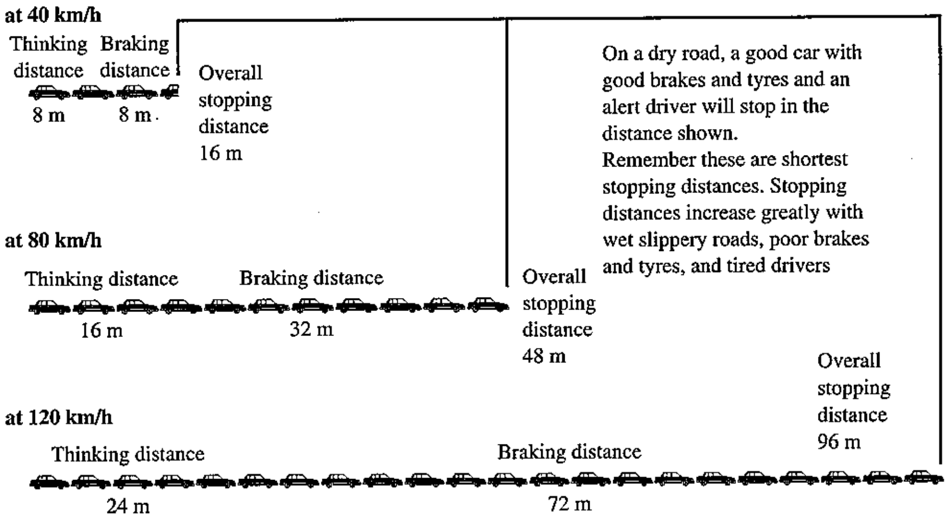


Diagram 3

Draw more shapes of your own design and record the values for p, i and A in a table. Make some of your shapes more difficult like the one in Diagram 3.

Can you find a formula connecting p, i and A?
[Hint: $\frac{1}{2}i$, $\frac{1}{2}p$?]
Try out your formula with some more shapes to see if it always works.

29. Stopping distances



This diagram from the Highway Code gives the overall stopping distances for cars travelling at various speeds.

What is meant by 'thinking distance'?
Work out the thinking distance for a car travelling at a speed of 90 km/h. What is the formula which connects the speed of the car and the thinking distance?

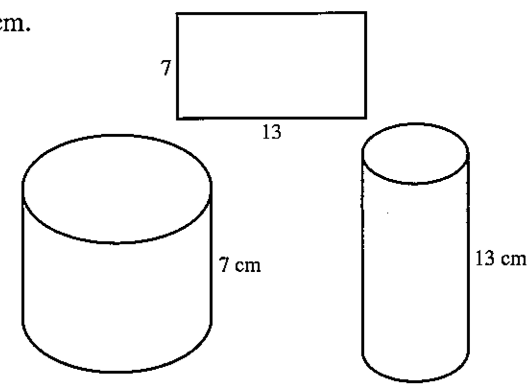
(More difficult)
Try to find a formula which connects the speed of the car and the *overall* stopping distance. It may help if you draw a graph of speed (across the page) against *braking* distance (up the page).
What curve are you reminded of?

Check that your formula gives the correct answer for the overall stopping distance at a speed of:
(a) 40 km/h (b) 120 km/h.

30. Maximum cylinder

A rectangular piece of paper has a fixed perimeter of 40 cm.
It could for example be 7 cm × 13 cm.
This paper can make a hollow cylinder of height 7 cm or of height 13 cm.
Work out the volume of each cylinder.

What dimensions should the paper have so that it can make a cylinder of the maximum possible volume?



11.2 Practical problems

1. Timetabling

(a) Every year a new timetable has to be written for the school. We will look at the problem of writing the timetable for one department (mathematics). The department allocates the teaching periods as follows:

Upper 6	2 sets (at the same times); 8 periods in 4 doubles.
Lower 6	2 sets (at the same times); 8 periods in 4 doubles.
Year 5	6 sets (at the same times); 5 single periods.
Year 4	6 sets (at the same times); 5 single periods.
Year 3	6 sets (at the same times); 5 single periods.
Year 2	6 sets (at the same times); 5 single periods.
Year 1	5 mixed ability forms; 5 single periods not necessarily at the same times.

Here are the teachers and the maximum number of maths periods which they can teach.

A	33	
B	33	
C	33	
D	20	
E	20	
F	15	(must be years 5, 4, 3)
G	10	(must be years 2, 1)
H	10	(must be years 2, 1)
I	5	(must be year 3)

Furthermore, to ensure some continuity of teaching, teachers B and C must teach the U6 (Upper Sixth) and teachers A, B, C, D, E, F must teach year 5.

Here is a timetable form which has been started:

M	5				U6 B, C	U6 B, C		
Tu		5	U6 B, C	U6 B, C				
W					5			
Th						5	U6 B, C	U6 B, C
F	U6 B, C	U6 B, C		5				

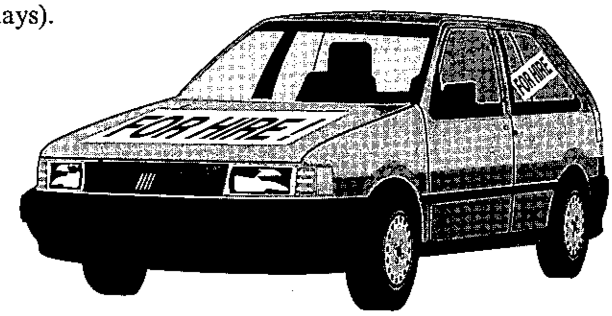
Your task is to write a complete timetable for the mathematics department subject to the restrictions already stated.

(b) If that was too easy, here are some changes.
U6 and L6 have 4 sets each (still 8 periods)
Two new teachers: J 20 periods maximum
K 15 periods maximum but cannot teach on Mondays.
Because of games lessons: A cannot teach Wednesday afternoon
B cannot teach Tuesday afternoon
C cannot teach Friday afternoon
Also: A, B, C and E must teach U6
A, B, C, D, E, F must teach year 5
For the pupils, games afternoons are as follows:
Monday year 2; Tuesday year 3; Wednesday 5 L6, U6;
Thursday year 4; Friday year 1.

2. Hiring a car

You are going to hire a car for one week (seven days).

Which of the firms below should you choose?



Gibson car hire	Snowdon rent-a-car	Hav-a-car
\$170 per week no charge up to 10 000 km	\$10 per day 6·5c per km	\$60 per week 500 km without charge 22c per km over 500 km

Work out as detailed an answer as possible.

3. Running a business

Mr Singh runs a small business making two sorts of steam cleaner: the basic model B and the deluxe model D.
Here are the details of the manufacturing costs:

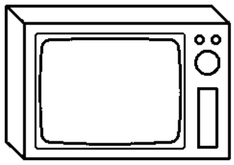
	model B	model D
Assembly time (in man-hours)	20 hours	30 hours
Component costs	\$35	\$25
Selling price	\$195	\$245

He employs 10 people and pays them each \$160 for a 40-hour week. He can spend up to \$525 per week on components.

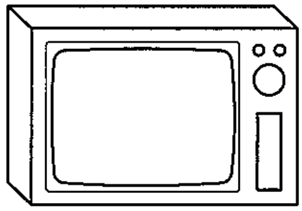
(a) In one week the firm makes and sells six cleaners of each model. Does he make a profit?
[Remember he has to pay his employees for a full week.]
(b) What number of each model should he make so that he makes as much profit as possible? Assume he can sell all the machines which he makes.

4. How many of each?

A shop owner has room in her shop for up to 20 televisions. She can buy either type A for $150 each or type B for $300 each.
She has a total of $4500 she can spend and she must have at least 6 of each type in stock. She makes a profit of $80 on each television of type A and a profit of $100 on each of type B.



A cost $150



B cost $300

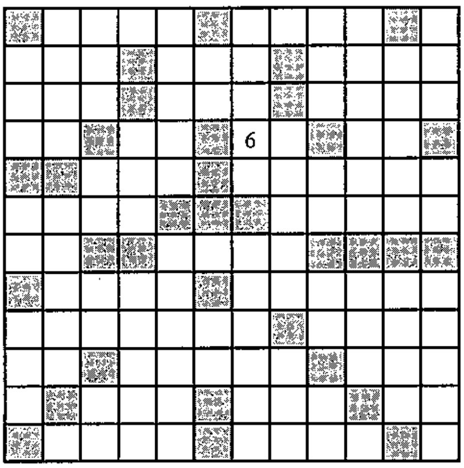
How many of each type should she buy so that she makes the maximum profit?

11.3 Puzzles and experiments

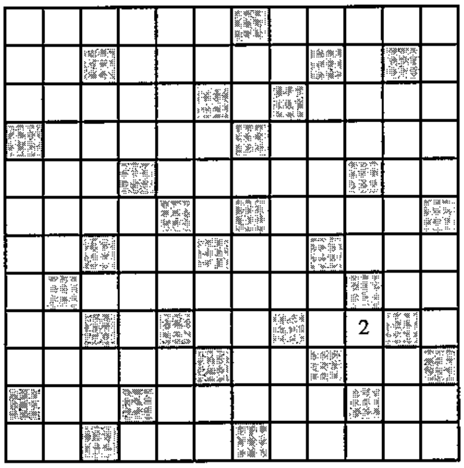
1. Cross numbers

(a) Copy out the cross number pattern.
(b) Fit all the given numbers into the correct spaces. Tick off the numbers from the lists as you write them in the square.

1.



2 digits	*3 digits*	*4 digits*	*5 digits*	*6 digits*
11	121	2104	14 700	216 841
17	147	2356	24 567	588 369
18	170	2456	25 921	846 789
19	174	3714	26 759	861 277
23	204	4711	30 388	876 452
31	247	5548	50 968	
37	287	5678	51 789	
58	324	6231	78 967	
61	431	6789	98 438	
62	450	7630		*7 digits*
62	612	9012		6 645 678
70	678	9921		
74	772			
81	774			
85	789			
94	870			
99				

2. 

2 digits		*3 digits*	*4 digits*	*5 digits*	*6 digits*
12	47	129	2096	12 641	324 029
14	48	143	3966	23 449	559 641
16	54	298	5019	33 111	956 782
18	56	325	5665	33 210	
20	63	331	6462	34 509	
21	67	341	7809	40 551	
23	81	443	8019	41 503	
26	90	831	8652	44 333	*7 digits*
27	91	923		69 786	1 788 932
32	93			88 058	5 749 306
38	98			88 961	
39	99			90 963	
46				94 461	
				99 654	

2. Estimating game

This is a game for two players. On squared paper draw an answer grid with the numbers shown below.

Answer grid

891	7047	546	2262	8526	429
2548	231	1479	357	850	7938
663	1078	2058	1014	1666	3822
1300	1950	819	187	1050	3393
4350	286	3159	442	2106	550
1701	4050	1377	4900	1827	957

The players now take turns to choose two numbers from the question grid below and multiply them on a calculator.

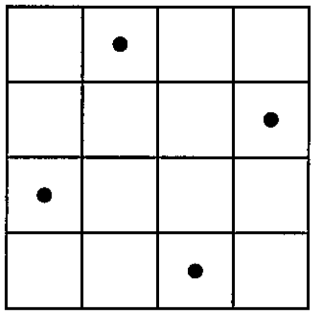
Question grid

11	26	81
17	39	87
21	50	98

The game continues until all the numbers in the answer grid have been crossed out. The object is to get four answers in a line (horizontally, vertically or diagonally). The winner is the player with most lines of four.
A line of *five* counts as *two* lines of four.
A line of *six* counts as *three* lines of four.

3. The chess board problem

(a) On the 4×4 square below we have placed four objects subject to the restriction that nowhere are there two objects on the same row, column or diagonal.



Subject to the same restrictions:

(i) find a solution for a 5×5 square, using five objects,
(ii) find a solution for a 6×6 square, using six objects,
(iii) find a solution for a 7×7 square, using seven objects,
(iv) find a solution for a 8×8 square, using eight objects.

It is called the chess board problem because the objects could be 'Queens' which can move any number of squares in any direction.

(b) Suppose we remove the restriction that no two Queens can be on the same row, column or diagonal. Is it possible to attack every square on an 8×8 chess board with less than eight Queens?
Try the same problem with other pieces like knights or bishops.

4. Creating numbers

Using only the numbers 1, 2, 3 and 4 once each and the operations $+, -, \times, \div, !$ create every number from 1 to 100.

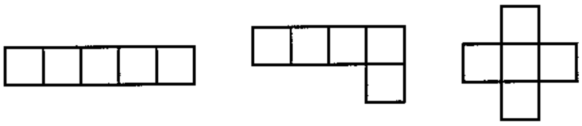
You can use the numbers as powers and you must use all of the numbers 1, 2, 3 and 4.

[4! is pronounced 'four factorial' and means $4 \times 3 \times 2 \times 1$ (i.e. 24)
similarly $3! = 3 \times 2 \times 1 = 6$
$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$]

Examples:
$1 = (4 - 3) \div (2 - 1)$
$20 = 4^2 + 3 + 1$
$68 = 34 \times 2 \times 1$
$100 = (4! + 1)(3! - 2!)$

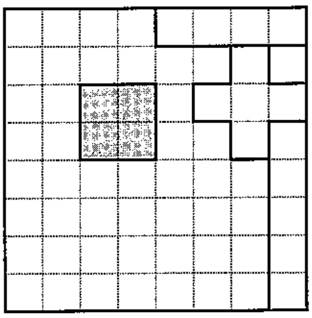
5. Pentominoes

A pentomino is a set of five squares joined along their edges. Here are three of the twelve different pentomino designs.



(a) Find the other nine pentomino designs to make up the complete set of twelve. Reflections or rotations of other pentominoes are not allowed.

(b) On squared paper draw an 8×8 square. It is possible to fill up the 8×8 square with the twelve different pentominoes together with a 2×2 square. Here we have made a possible start.



There are in fact many different ways in which this can be done.

(c) Now draw a 10×6 rectangle.
Try to fill up the rectangle with as many different pentominoes as you can. This problem is more difficult than the previous one but it is possible to fill up the rectangle with the twelve different pentominoes.

6. Calculator words

On a calculator work out $9508^2 + 192^2 + 10^2 + 6$. If you turn the calculator upside down and use a little imagination, you can see the word 'HEDGEHOG'.
Find the words given by the clues below.

1. $19 \times 20 \times 14 - 2{\cdot}66$ (not an upstanding man)
2. $(84 + 17) \times 5$ (dotty message)
3. $904^2 + 89\,621\,818$ (prickly customer)
4. $(559 \times 6) + (21 \times 55)$ (what a surprise!)
5. $566 \times 711 - 23\,617$ (bolt it down)
6. $\dfrac{9999 + 319}{8{\cdot}47 + 2{\cdot}53}$ (sit up and plead)
7. $\dfrac{2601 \times 6}{4^2 + 1^2}$; $(401 - 78) \times 5^2$ (two words) (not a great man)

8. $0{\cdot}4^2 - 0{\cdot}1^2$ (little Sidney)
9. $\dfrac{(27 \times 2000 - 2)}{(0{\cdot}63 \div 0{\cdot}09)}$ (not quite a mountain)
10. $(5^2 - 1^2)^4 - 14\,239$ (just a name)
11. $48^4 + 102^2 - 4^2$ (pursuits)
12. $615^2 + (7 \times 242)$ (almost a goggle)
13. $(130 \times 135) + (23 \times 3 \times 11 \times 23)$ (wobbly)
14. $164 \times 166^2 + 734$ (almost big)
15. $8794^2 + 25 \times 342{\cdot}28 + 120 \times 25$ (thin skin)
16. $0{\cdot}08 - (3^2 \div 10^4)$ (ice house)
17. $235^2 - (4 \times 36{\cdot}5)$ (shiny surface)
18. $(80^2 + 60^2) \times 3 + 81^2 + 12^2 + 3013$ (ship gunge)
19. $3 \times 17 \times (329^2 + 2 \times 173)$ (unlimited)
20. $230 \times 230\frac{1}{2} + 30$ (fit feet)
21. $33 \times 34 \times 35 + 15 \times 3$ (beleaguer)
22. $0{\cdot}32^2 + \frac{1}{1000}$ (Did he or didn't he?)
23. $(23 \times 24 \times 25 \times 26) + (3 \times 11 \times 10^3) - 20$ (help)
24. $(16^2 + 16)^2 - (13^2 - 2)$ (slander)
25. $(3 \times 661)^2 - (3^6 + 22)$ (pester)
26. $(22^2 + 29{\cdot}4) \times 10$; $(3{\cdot}03^2 - 0{\cdot}02^2) \times 100^2$ (four words) (Goliath)
27. $1{\cdot}25 \times 0{\cdot}2^6 + 0{\cdot}2^2$ (tissue time)
28. $(710 + (1823 \times 4)) \times 4$ (liquor)
29. $(3^3)^2 + 2^2$ (wriggler)
30. $14 + (5 \times (83^2 + 110))$ (bigger than a duck)
31. $2 \times 3 \times 53 \times 10^4 + 9$ (opposite to hello, almost!)
32. $(177 \times 179 \times 182) + (85 \times 86) - 82$ (good salesman)
33. $14^4 - 627 + 29$ (good book, by God!)
34. $6{\cdot}2 \times 0{\cdot}987 \times 1\,000\,000 - 860^2 + 118$ (flying ace)
35. $(426 \times 474) + (318 \times 487) + 22\,018$ (close to a bubble)