10 Statistics and Probability

Blaise Pascal (1623–1662) suffered the most appalling ill-health throughout his short life. He is best known for his work with Fermat on probability. This followed correspondence with a gentleman gambler who was puzzled as to why he lost so much in betting on the basis of the appearance of a throw of dice. Pascal's work on probability became of enormous importance and showed for the first time that absolute certainty is not a necessity in mathematics and science. He also studied physics, but his last years were spent in religious meditation and illness.

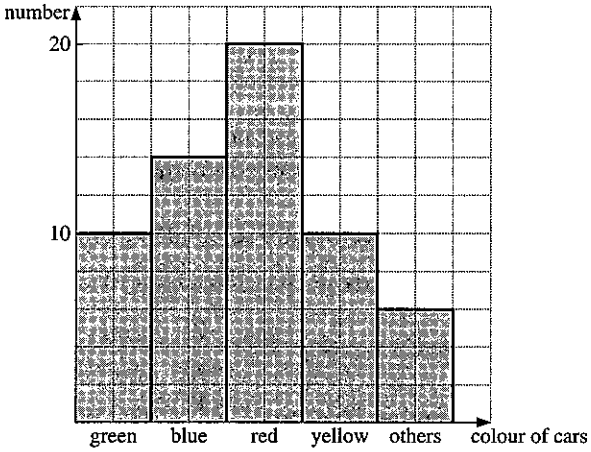
33 Construct and read bar charts, histograms, scatter diagrams and cumulative frequency diagrams; calculate the mean, median and mode

34 Calculate the probability of a single event, and simple combined events

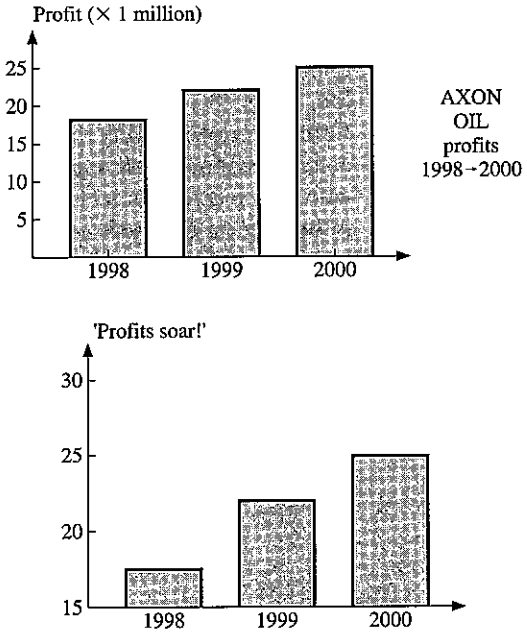
10.1 Data display

Bar chart

The length of each bar represents the quantity in question. The width of each bar has no significance. In the bar chart below, the number of the cars of each colour in a car park is shown. The bars can be joined together or separated.

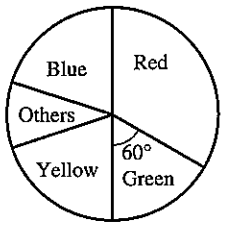


Example



The bar charts show the profits of 'AXON OIL' over a 3-year period.
The second graph has been drawn to give the impression that the profits have increased dramatically over the last three years.
How has this been done?

Pie chart



The information is displayed using sectors of a circle
This pie chart shows the same information as the bar chart on the previous page.
The angles of the sectors are calculated as follows:

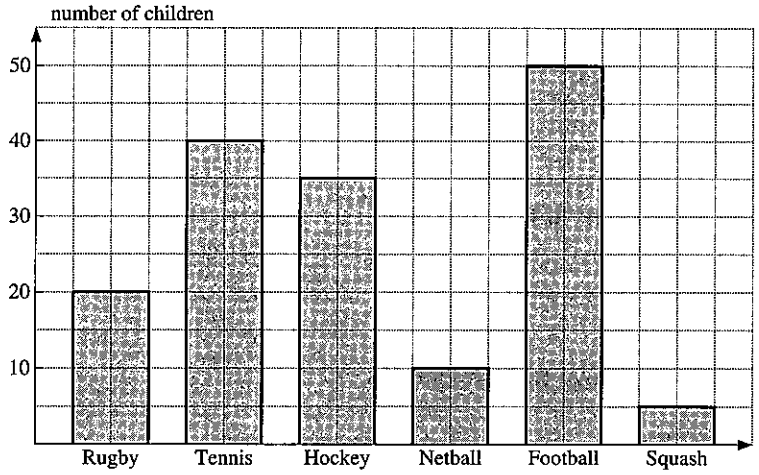
Total number of cars $= 10 + 14 + 20 + 10 + 6 = 60$

Angle representing green cars $= \frac{10}{60} \times 360° = 60°$

Angle representing blue cars $= \frac{14}{60} \times 360°$, etc.

Exercise 1

1. The bar chart shows the number of children playing various games on a given day.

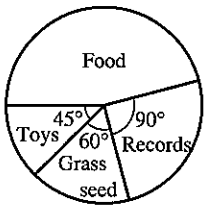


   (a) Which game had the least number of players?
   (b) What was the total number of children playing all the games?
   (c) How many more footballers were there than tennis players?

2. The table shows the number of cars of different makes in a car park. Illustrate this data on a bar chart.

Make	Fiat	Renault	Vauxhall	Rolls Royce	Ford	Datsun
Number	14	23	37	5	42	18

3. The pie chart illustrates the values of various goods sold by a certain shop. If the total value of the sales was \$24 000, find the sales value of:
   (a) toys
   (b) grass seed
   (c) records
   (d) food.



4. The table shows the colours of a random selection of 'Smarties'. Calculate the angles on a pie chart corresponding to each colour.

colour	red	green	blue	yellow	pink
number	5	7	11	4	9

5. A quantity of scrambled eggs is made using the following recipe:

ingredient	eggs	milk	butter	cheese	salt/pepper
mass	450 g	20 g	39 g	90 g	1 g

   Calculate the angles on a pie chart corresponding to each ingredient.

6. Calculate the angles on a pie chart corresponding to quantities A, B, C, D and E given in the tables.

(a)

quantity	A	B	C	D	E
number	3	5	3	7	0

(b)

quantity	A	B	C	D	E
mass	10 g	15 g	34 g	8 g	5 g

(c)

quantity	A	B	C	D	E
length	7	11	9	14	11

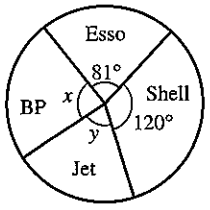
7. A firm making artificial sand sold its products in four countries:
5% were sold in Spain
15% were sold in France
15% were sold in Germany
65% were sold in U.K.
What would be the angles on a pie chart drawn to represent this information?

8. The weights of A, B, C are in the ratio 2 : 3 : 4. Calculate the angles representing A, B and C on a pie-chart.

9. The cooking times for meals L, M and N are in the ratio $3 : 7 : x$. On a pie-chart, the angle corresponding to L is 60°. Find x.

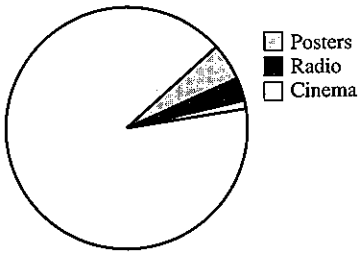
10. The results of an opinion poll of 2000 people are represented on a pie chart. The angle corresponding to 'don't know' is 18°. How many people in the sample did not know?

11. The pie chart illustrates the sales of various makes of petrol.
(a) What percentage of sales does 'Esso' have?
(b) If 'Jet' accounts for $12\frac{1}{2}$% of total sales, calculate the angles x and y.



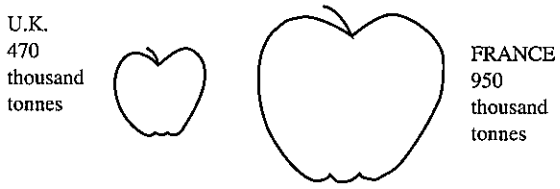
12.

	% of total spent
Press	51
Television	40
Posters	
Cinema	
Radio	3
Total	100



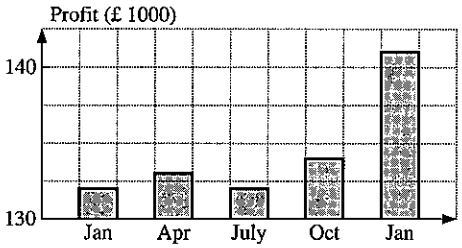
In Spain money was spent on advertisements in 1999 in the press, TV, posters, etc. The incomplete table and pie-chart show the way this was divided between the media.
(a) Calculate the angle of the sector representing television, and complete the pie-chart.
(b) The angle of the sector representing posters is 18°. Calculate the percentage spent on posters, and hence complete the table.

13. The diagram illustrates the production of apples in two countries.



In what way could the pictorial display be regarded as misleading?

14. The graph shows the performance of a company in the year in which a new manager was appointed. In what way is the graph misleading?



Histograms

In a histogram, the frequency of the data is shown by the *area* of each bar. Histograms resemble bar charts but are not to be confused with them: in bar charts the frequency is shown by the height of each bar. Histograms often have bars of varying widths. Because the area of the bar represents frequency, the height must be adjusted to correspond with the width of the bar. The vertical axis is not labelled frequency but frequency density.

$$\text{frequency density} = \frac{\text{frequency}}{\text{class width}}$$

Histograms can be used to represent both discrete data and continuous data, but their main purpose is for use with continuous data.

Example

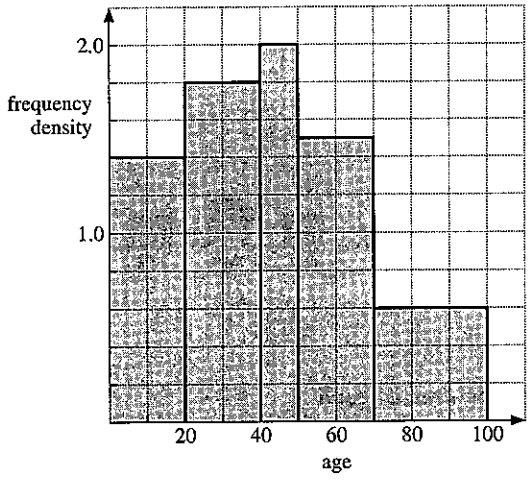
Draw a histogram from the table shown for the distribution of ages of passengers travelling on a flight to New York.

Note that the data has been collected into class intervals of different widths.

ages	frequency
$0 \leqslant x < 20$	28
$20 \leqslant x < 40$	36
$40 \leqslant x < 50$	20
$50 \leqslant x < 70$	30
$70 \leqslant x < 100$	18

To draw the histogram, the heights of the bars must be adjusted by calculating frequency density.

ages	frequency	frequency density (f.d.)
$0 \leqslant x < 20$	28	$28 \div 20 = 1{\cdot}4$
$20 \leqslant x < 40$	36	$36 \div 20 = 1{\cdot}8$
$40 \leqslant x < 50$	20	$20 \div 10 = 2$
$50 \leqslant x < 70$	30	$30 \div 20 = 1{\cdot}5$
$70 \leqslant x < 100$	18	$18 \div 30 = 0{\cdot}6$

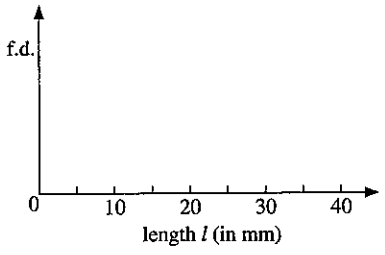


Exercise 2

1. The lengths of 20 copper nails were measured. The results are shown in the frequency table.

length l (in mm)	frequency	frequency density (f.d.)
$0 \leqslant L < 20$	5	$5 \div 20 = 0{\cdot}25$
$20 \leqslant L < 25$	5	
$25 \leqslant L < 30$	7	
$30 \leqslant L < 40$	3	

Calculate the frequency densities and draw the histogram as started below.



2. The volumes of 55 containers were measured and the results presented in a frequency table as shown in the table.

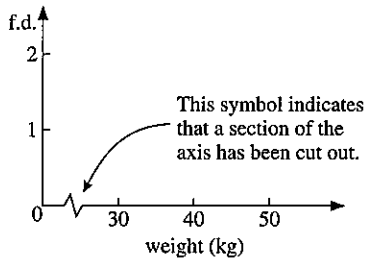
volume (mm^3)	frequency
$0 \leqslant V < 5$	5
$5 \leqslant V < 10$	3
$10 \leqslant V < 20$	12
$20 \leqslant V < 30$	17
$30 \leqslant V < 40$	13
$40 \leqslant V < 60$	5

Calculate the frequency densities and draw the histogram.

3. Thirty students in a class are weighed on the first day of term. Draw a histogram to represent this data.

weight (kg)	frequency
30–40	5
40–45	7
45–50	10
50–55	5
55–70	3

Note that the weights do not start a zero. This can be shown on the graph as follows:



4. The ages of 120 people passing through a turnstyle were recorded and are shown in the frequency table.

age (yrs)	frequency
–10	18
–15	46
–20	35
–30	13
–40	8

The notation –10 means '0 < age $\leqslant$ 10' and similarly –15 means '10 < age $\leqslant$ 15'. The class boundaries are 0, 10, 15, 20, 30, 40. Draw the histogram for the data.

5. Another common notation is used here for the masses of plums picked in an orchard, shown in the table below.

mass (g)	20–	30–	40–	60–	80–
frequency	11	18	7	5	0

The notation 20– means $20\,g \leqslant \text{mass} < 30\,g$.
Draw a histogram with class boundaries at 20, 30, 40, 60, 80.

6. The heights of 50 Olympic athletes were measured as shown in the table.

height (cm)	170–174	175–179	180–184	185–194
frequency	8	17	14	11

These values were rounded off to the nearest cm. For example, an athlete whose height h is 181 cm could be entered anywhere in the class $180{\cdot}5\,cm \leqslant h < 181{\cdot}5\,cm$. So the table is as follows:

height	169·5–174·5	174·5–179·5	179·5–184·5	184·5–194·5
frequency	8	17	14	11

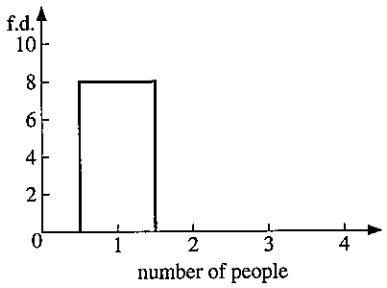
Draw a histogram with class boundaries at 169·5, 174·5, 179·5, ...

7. The number of people travelling in 33 vehicles one day was as shown in the table below.

number of people	1	2	3	4	5–6	7–10
frequency	8	11	6	4	2	2

In this case, the data is discrete. To represent this information on a histogram, draw the column for the value 2, for example, from 1·5 to 2·5, and that for the values 5–6 from 4·5 to 6·5 as shown below.

number of people	frequency	interval on histogram	width of interval	frequency density
1	8	0·5–1·5	1	8
2	11	1·5–2·5		
3	6			
4	4			
5–6	2	4·5–6·5	2	1
7–10	2			



Copy and complete the above table and the histogram which has been started on the right.

10.2 Mean, median and mode

(a) The *mean* of a series of numbers is obtained by adding the numbers and dividing the result by the number of numbers.
(b) The *median* of a series of numbers is obtained by arranging the numbers in ascending order and then choosing the number in the 'middle'. If there are *two* 'middle' numbers the median is the average (mean) of these two numbers.
(c) The *mode* of a series of numbers is simply the number which occurs most often.

Example

Find the mean, median and mode of the following numbers:
5, 4, 10, 3, 3, 4, 7, 4, 6, 5.

(a) Mean $= \dfrac{(5+4+10+3+3+4+7+4+6+5)}{10} = \dfrac{51}{10} = 5{\cdot}1$

(b) Median: arranging numbers in order of size
3, 3, 4, 4, 4, 5, 5, 6, 7, 10
↑
The median is the 'average' of 4 and 5
$\therefore$ median $= 4{\cdot}5$

(c) Mode = 4 (there are more 4's than any other number).

Frequency tables

A frequency table shows a number x such as a mark or a score, against the frequency f or number of times that x occurs.

The next example shows how these symbols are used in calculating the mean, the median and the mode.

The symbol Σ (or sigma) means 'the sum of'.

Example

The marks obtained by 100 students in a test were as follows:

mark (x)	0	1	2	3	4
frequency (f)	4	19	25	29	23

Find:
(a) the mean mark (b) the median mark (c) the modal mark

(a) Mean $= \dfrac{\Sigma xf}{\Sigma f}$

where Σxf means 'the sum of the products'
i.e. Σ(number $\times$ frequency)

and Σf means 'the sum of the frequencies'

$$\text{Mean} = \frac{(0 \times 4) + (1 \times 19) + (2 \times 25) + (3 \times 29) + (4 \times 23)}{100}$$

$$= \frac{248}{100} = 2{\cdot}48$$

(b) The median mark is the number between the 50th and 51st numbers. By inspection, both the 50th and 51st numbers are 3.

$\therefore$ Median = 3 marks

(c) The modal mark = 3

Exercise 3

1. Find the mean, median and mode of the following sets of numbers:
   (a) 3, 12, 4, 6, 8, 5, 4
   (b) 7, 21, 2, 17, 3, 13, 7, 4, 9, 7, 9
   (c) 12, 1, 10, 1, 9, 3, 4, 9, 7, 9
   (d) 8, 0, 3, 3, 1, 7, 4, 1, 4, 4

2. Find the mean, median and mode of the following sets of numbers:
   (a) 3, 3, 5, 7, 8, 8, 8, 9, 11, 12, 12
   (b) 7, 3, 4, 10, 1, 2, 1, 3, 4, 11, 10, 4
   (c) -3, 4, 0, 4, -2, -5, 1, 7, 10, 5
   (d) 1, $\frac{1}{2}$, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{1}{4}$, 2, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$

3. The mean weight of five men is 76 kg. The weights of four of the men are 72 kg, 74 kg, 75 kg and 81 kg. What is the weight of the fifth man?

4. The mean length of 6 rods is 44·2 cm. The mean length of 5 of them is 46 cm. How long is the sixth rod?

5. (a) The mean of 3, 7, 8, 10 and x is 6. Find x.
   (b) The mean of 3, 3, 7, 8, 10, x and x is 7. Find x.

6. The mean height of 12 men is 1·70 m, and the mean height of 8 women is 1·60 m. Find:
   (a) the total height of the 12 men,
   (b) the total height of the 8 women,
   (c) the mean height of the 20 men and women.

7. The total weight of 6 rugby players is 540 kg and the mean weight of 14 ballet dancers is 40 kg. Find the mean weight of the group of 20 rugby players and ballet dancers.

8. The mean weight of 8 boys is 55 kg and the mean weight of a group of girls is 52 kg. The mean weight of all the children is 53·2 kg. How many girls are there?

9. For the set of numbers below, find the mean and the median.

1, 3, 3, 3, 4, 6, 99.

   Which average best describes the set of numbers?

10. In a history test, Andrew got 62%. For the whole class, the mean mark was 64% and the median mark was 59%. Which 'average' tells Andrew whether he is in the 'top' half or the 'bottom' half of the class?

11. The mean age of three people is 22 and their median age is 20. The range of their ages is 16. How old is each person?

12. A group of 50 people were asked how many books they had read in the previous year; the results are shown in the frequency table below. Calculate the mean number of books read per person.

Number of books	0	1	2	3	4	5	6	7	8
Frequency	5	5	6	9	11	7	4	2	1

13. A number of people were asked how many coins they had in their pockets; the results are shown below. Calculate the mean number of coins per person.

Number of coins	0	1	2	3	4	5	6	7
Frequency	3	6	4	7	5	8	5	2

14. The following tables give the distribution of marks obtained by different classes in various tests. For each table, find the mean, median and mode.

(a)

Mark	0	1	2	3	4	5	6
Frequency	3	5	8	9	5	7	3

(b)

Mark	15	16	17	18	19	20
Frequency	1	3	7	1	5	3

(c)

Mark	0	1	2	3	4	5	6
Frequency	10	11	8	15	25	20	11

15. One hundred golfers play a certain hole and their scores are summarised below.

Score	2	3	4	5	6	7	8
Number of players	2	7	24	31	18	11	7

Find:
(a) the mean score (b) the median score.

16. The number of goals scored in a series of football matches was as follows:

Number of goals	1	2	3
Number of matches	8	8	x

(a) If the mean number of goals is 2·04, find x.
(b) If the modal number of goals is 3, find the smallest possible value of x.
(c) If the median number of goals is 2, find the largest possible value of x.

17. In a survey of the number of occupants in a number of cars, the following data resulted.

Number of occupants	1	2	3	4
Number of cars	7	11	7	x

(a) If the mean number of occupants is $2\frac{1}{3}$, find x.
(b) If the mode is 2, find the largest possible value of x.
(c) If the median is 2, find the largest possible value of x.

18. The numbers 3, 5, 7, 8 and N are arranged in ascending order. If the mean of the numbers is equal to the median, find N.

19. The mean of 5 numbers is 11. The numbers are in the ratio 1 : 2 : 3 : 4 : 5. Find the smallest number.

20. The mean of a set of 7 numbers is 3·6 and the mean of a different set of 18 numbers is 5·1. Calculate the mean of the 25 numbers.

21. The results of 24 students in a test are given in the table.

mark	frequency
85–99	4
70–84	7
55–69	8
40–54	5

For grouped data such as this, each group can be represented *approximately* by its mid-point. For example, for the 85–99 interval, we say there are 4 marks of 92 (the mid-point).
(a) Find the mid-point of each group of marks and calculate an estimate of the mean mark.
(b) Explain why your answer is an estimate.

Scatter graphs

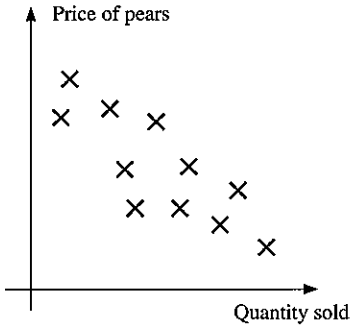
Sometimes it is important to discover if there is a connection or relationship between two sets of data.

Examples:

- Are more ice creams sold when the weather is hot?
- Do tall people have higher pulse rates?
- Are people who are good at maths also good at science?
- Does watching TV improve examination results?

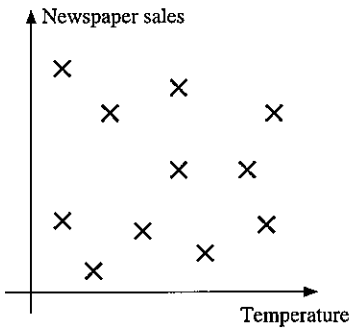
If there is a relationship, it will be easy to spot if your data is plotted on a scatter diagram – that is a graph in which one set of data is plotted on the horizontal axis and the other on the vertical axis.

Here is a scatter graph showing the price of pears and the quantity sold.



We can see a *connection* – when the price was high the sales were low and when the price went down the sales increased.

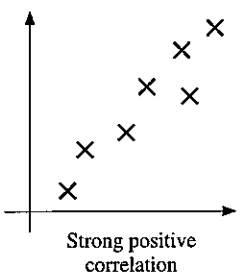
This scatter graph shows the sales of a newspaper and the temperature. We can see there is *no connection* between the two variables.

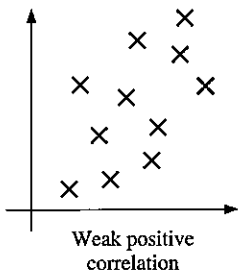


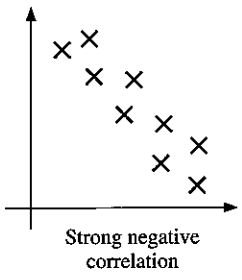
Correlation

The word correlation describes how things *co-relate*. There is correlation between two sets of data if there is a connection or relationship.

The correlation between two sets of data can be positive or negative and it can be strong or weak as indicated by the scatter graphs below.


Strong positive correlation


Weak positive correlation

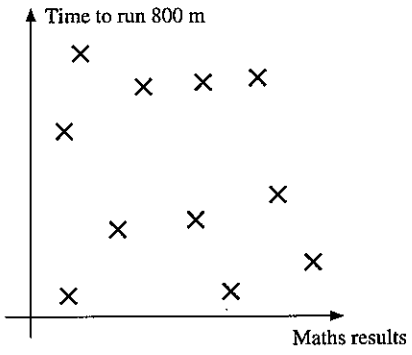

Strong negative correlation

When the correlation is positive the points are around a line which slopes upwards to the right. When the correlation is negative the 'line' slopes downwards to the right.

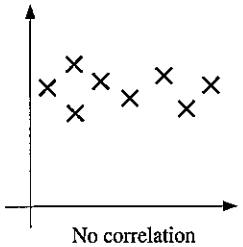
When the correlation is strong the points are bunched close to a line through their midst. When the correlation is weak the points are more scattered.

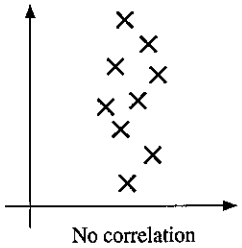
It is important to realise that often there is *no* correlation between two sets of data.

If, for example, we take a group of students and plot their maths test results against their time to run 800 m, the graph might look like the one on the right. A common mistake in this topic is to 'see' a correlation on a scatter graph where none exists.



There is also *no* correlation in these two scatter graphs.


No correlation


No correlation

Line of best fit

When a scatter graph shows either positive or negative correlation, a *line of best fit* can be drawn. The sums of the distances to points on either side of the line are equal and there should be an equal number of points on each side of the line. The line is easier to draw when a transparent ruler is used.

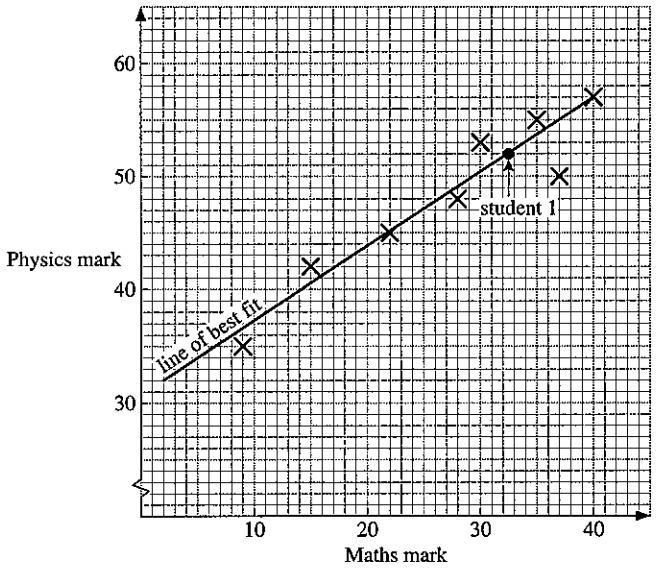
Here are the marks obtained in two tests by 9 students.

Student	A	B	C	D	E	F	G	H	I
Maths mark	28	22	9	40	37	35	30	23	?
Physics mark	48	45	34	57	50	55	53	45	52

A line of best fit can be drawn as there is strong positive correlation between the two sets of marks.

The line of best fit can be used to estimate the maths result of student 1, who missed the maths test but scored 52 in the physics test.

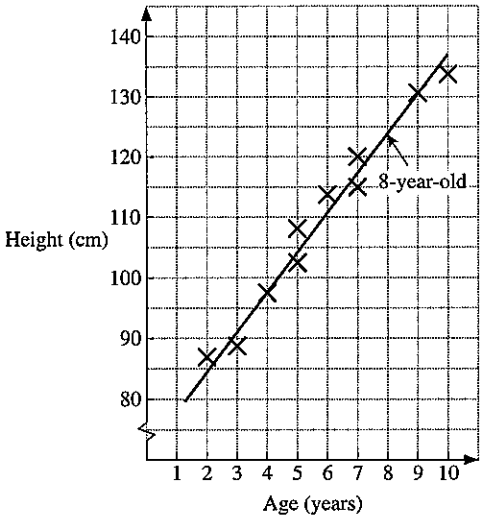
We can *estimate that student 1 would have scored about* 33 in the maths test. It is not possible to be *very* accurate using scatter graphs. It is reasonable to state that student 1 'might have scored between 30 and 36' in the maths test.



Here is a scatter graph in which the heights of boys of different ages is recorded. A line of best fit is drawn.

(a) We can estimate that the height of an 8-year-old boy might be about 123 cm [say between 120 and 126 cm].

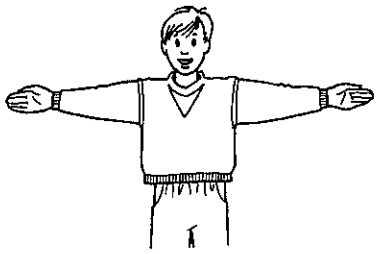
(b) We can only predict a height within the range of values plotted. We could not extend the line of best and use it to predict the height of a 30 year old! Why not?



Exercise 4

1. Make the following measurements for everyone in your class:

height	(nearest cm)
arm span	(nearest cm)
head circumference	(nearest cm)
hand span	(nearest cm)
pulse rate	(beats/minute)



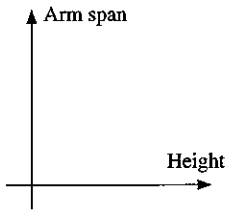
For greater consistency of measuring, one person (or perhaps two people) should do all the measurements of one kind (except on themselves!).

Enter all the measurements in a table, either on the board or on a sheet of paper.

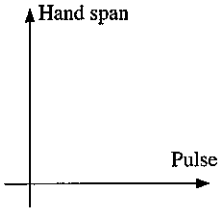
Name	Height	Armspan	Head
Roger	161	165	56
Liz	150	148	49
Gill			

(a) Draw the scatter graphs shown below:

(i)



(ii)



(b) Describe the correlation, if any, in the scatter graphs you drew in part (a).

(c) (i) Draw a scatter graph of two measurements where you think there might be positive correlation.
(ii) Was there indeed a positive correlation?

2. Plot the points given on a scatter graph, with s across the page and p up the page. Draw axes with values from 0 to 20.
Describe the correlation, if any, between the values of s and p.
[i.e. 'strong negative', 'weak positive' etc.]

(a)

s	7	16	4	12	18	6	20	4	10	13
p	8	15	6	12	17	9	18	7	10	14

(b)

s	3	8	12	15	16	5	6	17	9
p	4	2	10	17	5	10	17	11	15

(c)

s	11	1	16	7	2	19	8	4	13	18
p	5	12	7	14	17	1	11	8	11	5

In Questions 3, 4 and 5 plot the points given on a scatter graph,
with s across the page and p up the page.
draw axes with the values from 0 to 20.
If possible draw a line of best fit on the graph.
Where possible estimate the value of p on the line of best fit where $s = 10$.

3.

s	2	14	14	4	12	18	12	6
p	5	15	16	6	12	18	13	7

4.

s	2	15	17	3	20	3	6
p	13	7	5	12	4	13	11

5.

s	4	10	15	18	19	4	19	5
p	19	16	11	19	15	3	1	9

6. The following data gives the marks of 11 students in a French test and in a German test.

French	15	36	36	22	23	27	43	22	43	40	26
German	6	28	35	18	28	28	37	9	41	45	17

(a) Plot this data on a scatter graph, with French marks on the horizontal axis.
(b) Draw the line of best fit.
(c) Estimate the German mark of a student who got 30 in French.
(d) Estimate the French mark of a student who got 45 in German.

7. The data below gives the petrol consumption figures of cars, with the same size engine, when driven at different speeds.

Speed (m.p.h.)	30	62	40	80	70	55	75
Petrol consumption (m.p.g.)	38	25	35	20	26	34	22

(a) Plot a scatter graph and draw a line of best fit.
(b) Estimate the petrol consumption of a car travelling at 45 m.p.h.
(c) Estimate the speed of a car whose petrol consumption is 27 m.p.g.

10.3 Cumulative frequency

Cumulative frequency is the total frequency up to a given point.
A cumulative frequency curve (or ogive) shows the *median* at the 50th percentile of the cumulative frequency.
The value at the 25th percentile is known as the *lower quartile*, and that at the 75th percentile as the *upper quartile*.
A measure of the spread or dispersion of the data is given by the *interquartile range* where

inter-quartile range = upper quartile − lower quartile

Example

The marks obtained by 80 students in an examination are shown below.

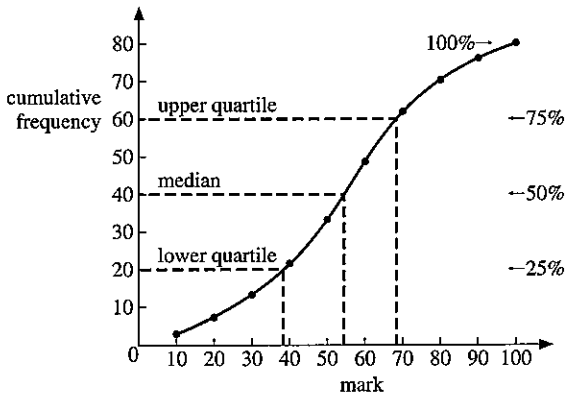
mark	frequency	cumulative frequency	marks represented by cumulative frequency
1–10	3	3	$\leqslant 10$
11–20	5	8	$\leqslant 20$
21–30	5	13	$\leqslant 30$
31–40	9	22	$\leqslant 40$
41–50	11	33	$\leqslant 50$
51–60	15	48	$\leqslant 60$
61–70	14	62	$\leqslant 70$
71–80	8	70	$\leqslant 80$
81–90	6	76	$\leqslant 90$
91–100	4	80	$\leqslant 100$

The table also shows the cumulative frequency.
Plot a cumulative frequency curve and hence estimate:
(a) the median
(b) the inter-quartile range.

The points on the graph are plotted at the upper limit of each group of marks.

From the cumulative frequency curve

median = 55 marks
lower quartile = 37·5 marks
upper quartile = 68 marks
$\therefore$ inter-quartile range = 68 − 37·5
= 30·5 marks.



Exercise 5

1. Figure 1 shows the cumulative frequency curve for the marks of 60 students in an examination.

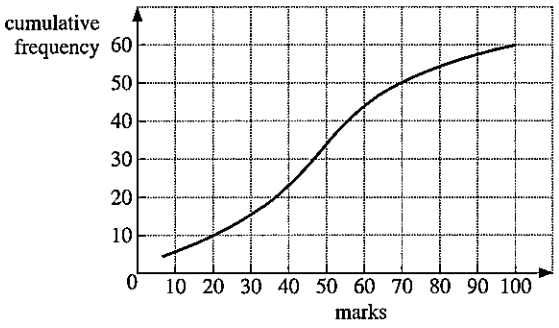


Figure 1

From the graph, estimate:
(a) the median mark,
(b) the mark at the lower quartile and at the upper quartile,
(c) the inter-quartile range,
(d) the pass mark if two-thirds of the students passed,
(e) the number of students achieving less than 40 marks.

2. Figure 2 shows the cumulative frequency curve for the marks of 140 students in an examination.

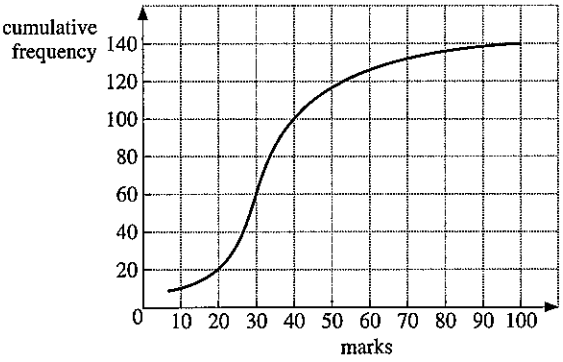


Figure 2

From the graph, estimate:
(a) the median mark,
(b) the mark at the lower quartile and at the upper quartile,
(c) the inter-quartile range,
(d) the pass mark if three-fifths of the students passed,
(e) the number of students achieving more than 0 marks.

In questions **3** to **6**, draw a cumulative frequency curve, and find:

(a) the median, (b) the interquartile range.

3.

mass (kg)	frequency
1–5	4
6–10	7
11–15	11
16–20	18
21–25	22
26–30	10
31–35	5
36–40	3

4.

length (cm)	frequency
41–50	6
51–60	8
61–70	14
71–80	21
81–90	26
91–100	14
101–110	7
111–120	4

5.

time (seconds)	frequency
36–45	3
46–55	7
56–65	10
66–75	18
76–85	12
86–95	6
96–105	4

6.

number of marks	frequency
1–10	0
11–20	2
21–30	4
31–40	10
41–50	17
51–60	11
61–70	3
71–80	3

7. In an experiment, 50 people were asked to guess the weight of a bunch of daffodils in grams. The guesses were as follows:

47	39	21	30	42	35	44	36	19	52
23	32	66	29	5	40	33	11	44	22
27	58	38	37	48	63	23	40	53	24
47	22	44	33	13	59	33	49	57	30
17	45	38	33	25	40	51	56	28	64

Construct a frequency table using intervals 0–9, 10–19, 20–29, etc. Hence draw a cumulative frequency curve and estimate:

(a) the median weight,

(b) the inter-quartile range,

(c) the number of people who guessed a weight within 10 grams of the median.

8. In a competition, 30 children had to pick up as many paper clips as possible in one minute using a pair of tweezers. The results were as follows:

3	17	8	11	26	23	18	28	33	38
12	38	22	50	5	35	39	30	31	43
27	34	9	25	39	14	27	16	33	49

Construct a frequency table using intervals 1–10, 11–20, etc. and hence draw a cumulative frequency curve.
(a) From the curve, estimate the median number of clips picked up.
(b) From the frequency table, estimate the mean of the distribution using the mid-interval values 5·5, 15·5, etc.
(c) Calculate the exact value of the mean using the original data.
(d) Why is it possible only to estimate the mean in part (b)?

9. The children in two schools took the same test in mathematics and their results are shown.

St Mary's School	Birchwood School
median mark = 52%	median mark = 51·8%
IQR = 7·2	IQR = 11·2

What can you say about these two sets of results?

Note: IQR is shorthand for inter-quartile range.

10. As part of a health improvement programme, people from one town and from one village in Gambia were measured. Here are the results.

People in town	People in village
median height = 171 cm	median height = 163 cm
IQR = 8·4	IQR = 3·7

What can you say about these two sets of results?

10.4 Simple probability

Probability theory is not the sole concern of people interested in betting, although it is true to say that a 'lucky' poker player is likely to be a player with a sound understanding of probability. All major airlines regularly overbook aircraft because they can usually predict with accuracy the probability that a certain number of passengers will fail to arrive for the flight.

Suppose a 'trial' can have n equally likely results and suppose that a 'success' can occur in s ways (from the n). Then the probability of a 'success' $= \frac{s}{n}$.

- If an event **cannot** happen the probability of it occurring is 0.
- If an event is **certain** to happen the probability of it occuring is 1.
- All probabilities lie between 0 and 1.
  You write probabilities using fractions or decimals.

Example 1

The numbers 1 to 20 are each written on a card.
The 20 cards are mixed together.
One card is chosen at random from the pack.
Find the probability that the number on the card is:

(a) even (b) a factor of 24 (c) prime.

We will use '$p(x)$' to mean 'the probability of x'.

(a) $p(\text{even}) = \frac{10}{20}$
$= \frac{1}{2}$

(b) $p(\text{factor of } 24)$
$= p(1, 2, 3, 4, 6, 8, 12)$
$= \frac{7}{20}$

(c) $p(\text{prime})$
$= p(2, 3, 5, 7, 11, 13, 17, 19)$
$= \frac{8}{20} = \frac{2}{5}$

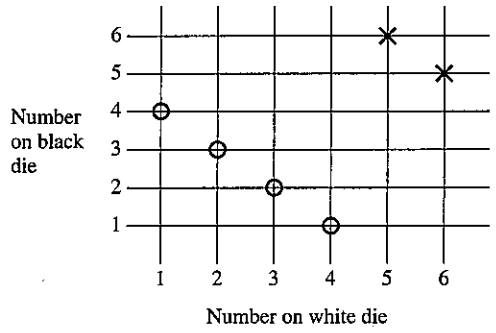
In each case, we have counted the number of ways in which a 'success' can occur and divided by the number of possible results of a 'trial'.

Example 2

A black die and a white die are thrown at the same time. Display all the possible outcomes. Find the probability of obtaining:

(a) a total of 5,
(b) a total of 11,
(c) a 'two' on the black die and a 'six' on the white die.

It is convenient to display all the possible outcomes on a grid.



There are 36 possible outcomes, shown where the lines cross.

(a) There are four ways of obtaining a total of 5 on the two dice. They are shown circled on the diagram.

$\therefore$ Probability of obtaining a total of $5 = \frac{4}{36}$

(b) There are two ways of obtaining a total of 11. They are shown with a cross on the diagram.

$\therefore$ p (total of 11) $= \frac{2}{36} = \frac{1}{18}$

(c) There is only one way of obtaining a 'two' on the black die and a 'six' on the white die.

$\therefore \quad p$ (2 on black and 6 on white) $= \frac{1}{36}$

Exercise 6

In this exercise, all dice are normal cubic dice with faces numbered 1 to 6.

1. A fair die is thrown once. Find the probability of obtaining:
(a) a six,
(b) an even number,
(c) a number greater than 3,
(d) a three or a five.

2. The two sides of a coin are known as 'head' and 'tail'. A 10c and a 5c coin are tossed at the same time. List all the possible outcomes. Find the probability of obtaining:
(a) two heads, (b) a head and a tail.

3. A bag contains 6 red balls and 4 green balls.
(a) Find the probability of selecting at random:
(i) a red ball (ii) a green ball.
(b) One red ball is removed from the bag. Find the new probability of selecting at random
(i) a red ball (ii) a green ball.

4. One letter is selected at random from the word 'UNNECESSARY'. Find the probability of selecting:
(a) an R (b) an E
(c) an O (d) a C

5. Three coins are tossed at the same time. List all the possible outcomes. Find the probability of obtaining:
(a) three heads,
(b) two heads and one tail,
(c) no heads,
(d) at least one head.

6. A bag contains 10 red balls, 5 blue balls and 7 green balls. Find the probability of selecting at random:
(a) a red ball,
(b) a green ball,
(c) a blue *or* a red ball,
(d) a red *or* a green ball.

7. Cards with the numbers 2 to 101 are placed in a hat. Find the probability of selecting:
(a) an even number,
(b) a number less than 14,
(c) a square number,
(d) a prime number less than 20.

8. A red die and a blue die are thrown at the same time. List all the possible outcomes in a systematic way. Find the probability of obtaining:
(a) a total of 10,
(b) a total of 12,
(c) a total less than 6,
(d) the same number on both dice,
(e) a total more than 9.
What is the most likely total?

9. A die is thrown; when the result has been recorded, the die is thrown a second time. Display all the possible outcomes of the two throws. Find the probability of obtaining:
(a) a total of 4 from the two throws,
(b) a total of 8 from the two throws,
(c) a total between 5 and 9 inclusive from the two throws,
(d) a number on the second throw which is double the number on the first throw,
(e) a number on the second throw which is four times the number on the first throw.

10. Find the probability of the following:
(a) throwing a number less than 8 on a single die,
(b) obtaining the same number of heads and tails when five coins are tossed,
(c) selecting a square number from the set
$A = \{4, 9, 16, 25, 36, 49\}$,
(d) selecting a prime number from the set A.

11. Four coins are tossed at the same time. List all the possible outcomes in a systematic way. Find the probability of obtaining:
(a) two heads and two tails,
(b) four tails,
(c) at least one tail,
(d) three heads and one tail.

Remember
'head' and 'tail' are the two sides of a coin.

12. Louise buys five raffle tickets out of 1000 sold. She does not win first prize. What is the probability that she wins second prize?

13. Tickets numbered 1 to 1000 were sold in a raffle for which there was one prize. Mr Kahn bought all the tickets containing at least one '3' because '3' was his lucky number. What was the probability of Mr Kahn winning?

14. One ball is selected at random from a bag containing 12 balls of which x are white.
(a) What is the probability of selecting a white ball?
When a further 6 white balls are added the probability of selecting a white ball is doubled.
(b) Find x.

15. Two dice and two coins are thrown at the same time. Find the probability of obtaining:
    (a) two heads and a total of 12 on the dice,
    (b) a head, a tail and a total of 9 on the dice,
    (c) two tails and a total of 3 on the dice.
    What is the most likely outcome?

16. A red, a blue and a green die are all thrown at the same time. Display all the possible outcomes in a suitable way. Find the probability of obtaining:
    (a) a total of 18 on the three dice,
    (b) a total of 4 on the three dice,
    (c) a total of 10 on the three dice,
    (d) a total of 15 on the three dice,
    (e) a total of 7 on the three dice,
    (f) the same number on each die.

10.5 Exclusive and independent events

Two events are *exclusive* if they cannot occur at the same time:
e.g. Selecting an 'even number' or selecting a 'one' from a set of numbers.

The 'OR' rule:

For exclusive events A and B

$$p(\text{A or B}) = p(\text{A}) + p(\text{B})$$

Two events are *independent* if the occurrence of one event is unaffected by the occurrence of the other.
e.g. Obtaining a 'head' on one coin, and a 'tail' on another coin when the coins are tossed at the same time.

The 'AND' rule:

$$p(\text{A } \textit{and } \text{B}) = p(\text{A}) \times p(\text{B})$$

where $p(\text{A})$ = probability of A occurring etc. This is the multiplication law.

Example 1

One ball is selected at random from a bag containing 5 red balls, 2 yellow balls and 4 white balls. Find the probability of selecting a red ball or a white ball.

The two events are exclusive.

$$\begin{aligned} p(\text{red ball } \textit{or} \text{ white ball}) &= p(\text{red}) + p(\text{white}) \\ &= \tfrac{5}{11} + \tfrac{4}{11} \\ &= \tfrac{9}{11} \end{aligned}$$

Example 2

A fair coin is tossed and a fair die is rolled. Find the probability of obtaining a'head' and a 'six'.

The two events are independent

$$\begin{aligned} p(\text{head } \textit{and } \text{six}) &= p(\text{head}) \times p(\text{six}) \\ &= \tfrac{1}{2} \times \tfrac{1}{6} \\ &= \tfrac{1}{12} \end{aligned}$$

Exercise 7

1. A coin is tossed and a die is thrown. Write down the probability of obtaining:
   (a) a 'head' on the coin,
   (b) an odd number on the die,
   (c) a 'head' on the coin and an odd number on the die.

2. A ball is selected at random from a bag containing 3 red balls, 4 black balls and 5 green balls. The first ball is replaced and a second is selected. Find the probability of obtaining:
   (a) two red balls, (b) two green balls.

3. The letters of the word 'INDEPENDENT' are written on individual cards and the cards are put into a box. A card is selected and then replaced and then a second card is selected. Find the probability of obtaining:
   (a) the letter 'P' twice, (b) the letter 'E' twice.

4. Three coins are tossed and two dice are thrown at the same time. Find the probability of obtaining:
   (a) three heads and a total of 12 on the dice,
   (b) three tails and a total of 9 on the dice.

5. When a golfer plays any hole, he will take 3, 4, 5, 6, or 7 strokes with probabilities of $\frac{1}{10}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{1}{5}$ and $\frac{1}{10}$ respectively. He never takes more than 7 strokes. Find the probability of the following events:
   (a) scoring 4 on each of the first three holes,
   (b) scoring 3, 4 and 5 (in that order) on the first three holes,
   (c) scoring a total of 28 for the first four holes,
   (d) scoring a total of 10 for the first three holes,
   (e) scoring a total of 20 for the first three holes.

6. A coin is biased so that it shows 'heads' with a probability of $\frac{2}{3}$. The same coin is tossed three times. Find the probability of obtaining:
   (a) two tails on the first two tosses,
   (b) a head, a tail and a head (in that order),
   (c) two heads and one tail (in any order).

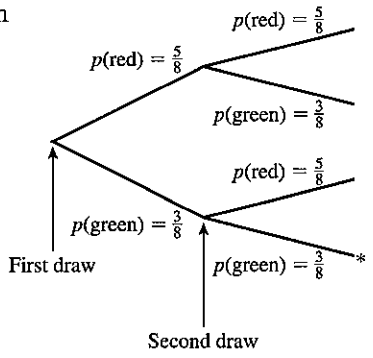
10.6 Tree diagrams

Example 1

A bag contains 5 red balls and 3 green balls. A ball is drawn at random and then replaced. Another ball is drawn.
What is the probability that both balls are green?

The branch marked * involves the selection of a green ball twice.
The probability of this event is obtained by simply multiplying the fractions on the two branches.

$\therefore \quad p(\text{two green balls}) = \frac{3}{8} \times \frac{3}{8} = \frac{9}{64}$

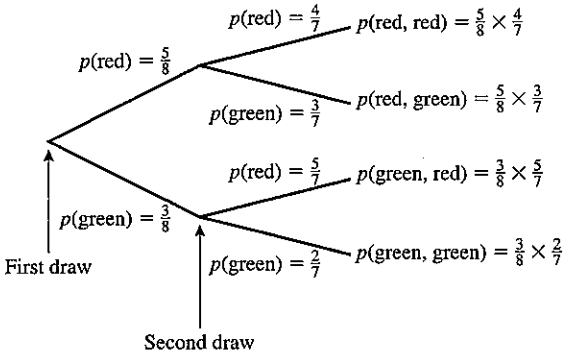


Example 2

A bag contains 5 red balls and 3 green balls. A ball is selected at random and not replaced. A second ball is then selected. Find the probability of selecting:
(a) two green balls
(b) one red ball and one green ball.

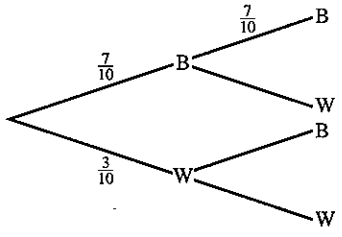
(a) $p(\text{two green balls}) = \frac{3}{8} \times \frac{2}{7}$
$= \frac{3}{28}$

(b) $p(\text{one red, one green}) = (\frac{5}{8} \times \frac{3}{7}) + (\frac{3}{8} \times \frac{5}{7})$
$= \frac{15}{28}$



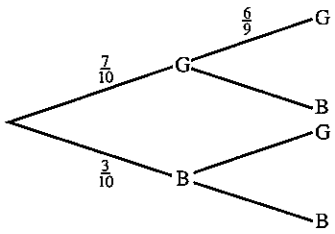
Exercise 8

1. A bag contains 10 discs; 7 are black and 3 white. A disc is selected, and then replaced. A second disc is selected. Copy and complete the tree diagram showing all the probabilities and outcomes.
   Find the probability of the following:
   (a) both discs are black, (b) both discs are white.



2. A bag contains 5 red balls and 3 green balls. A ball is drawn and then replaced before a ball is drawn again. Draw a tree diagram to show all the possible outcomes. Find the probability that:
(a) two green balls are drawn,
(b) the first ball is red and the second is green.

3. A bag contains 7 green discs and 3 blue discs. A disc is drawn and *not* replaced.
A second disc is drawn. Copy and complete the tree diagram.
Find the probability that:
(a) both discs are green,
(b) both discs are blue.



4. A bag contains 5 red balls, 3 blue balls and 2 yellow balls. A ball is drawn and not replaced. A second ball is drawn. Find the probability of drawing:
(a) two red balls,
(b) one blue ball and one yellow ball,
(c) two yellow balls,
(d) two balls of the same colour.

5. A bag contains 4 red balls, 2 green balls and 3 blue balls. A ball is drawn and not replaced. A second ball is drawn. Find the probability of drawing:
(a) two blue balls,
(b) two red balls,
(c) one red ball and one blue ball,
(d) one green ball and one red ball.

6. A six-sided die is thrown three times. Draw a tree diagram, showing at each branch the two events: 'six' and 'not six'. What is the probability of throwing a total of:
(a) three sixes,
(b) no sixes,
(c) one six,
(d) at least one six (use part (b)).

7. A bag contains 6 red marbles and 4 blue marbles. A marble is drawn at random and not replaced. Two further draws are made, again without replacement. Find the probability of drawing:
(a) three red marbles, (b) three blue marbles,
(c) no red marbles, (d) at least one red marble.

8. When a cutting is taken from a geranium the probability that it grows is $\frac{3}{4}$. Three cuttings are taken. What is the probability that:
(a) all three grow,
(b) none of them grow?

9. A die has its six faces marked 0, 1, 1, 1, 6, 6. Two of these dice are thrown together and the total score is recorded. Draw a tree diagram.
(a) How many different totals are possible?
(b) What is the probability of obtaining a total of 7?

10. A coin is biased so that the probability of a 'head' is $\frac{3}{4}$. Find the probability that, when tossed three times, it shows:
(a) three tails,
(b) two heads and one tail,
(c) one head and two tails,
(d) no tails.
Write down the sum of the probabilities in (a), (b), (c) and (d).

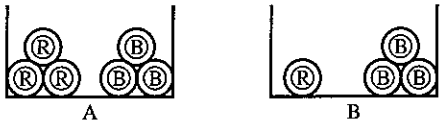
11. A teacher decides to award exam grades A, B or C by a new method. Out of 20 children, three are to receive A's, five B's and the rest C's. She writes the letters A, B and C on 20 pieces of paper and invites the pupils to draw their exam result, going through the class in alphabetical order. Find the probability that:
(a) the first three pupils all get grade 'A'.
(b) the first three pupils all get grade 'B',
(c) the first three pupils all get different grades,
(d) the first four pupils all get grade B.
(Do not cancel down the fractions.)

12. The probability that an amateur golfer actually hits the ball is (regrettably for all concerned) only $\frac{1}{10}$. If four separate attempts are made, find the probability that the ball will be hit:
(a) four times,
(b) at least twice,
(c) not at all.

13. A box contains x milk chocolates and y plain chocolates. Two chocolates are selected at random. Find, in terms of x and y, the probability of choosing:
(a) a milk chocolate on the first choice,
(b) two milk chocolates,
(c) one of each sort,
(d) two plain chocolates.

14. If a hedgehog crosses a certain road before 7.00 a.m., the probability of being run over is $\frac{1}{10}$. After 7.00 a.m., the corresponding probability is $\frac{3}{4}$. The probability of the hedgehog waking up early enough to cross before 7.00 a.m., is $\frac{4}{5}$.
What is the probability of the following events:
(a) the hedgehog waking up too late to reach the road before 7.00 a.m.,
(b) the hedgehog waking up early and crossing the road in safety,
(c) the hedgehog waking up late and crossing the road in safety,
(d) the hedgehog waking up early and being run over,
(e) the hedgehog crossing the road in safety.

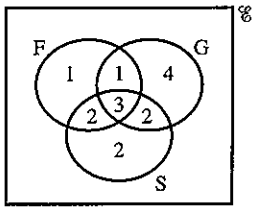
15. Bag A contains 3 red balls and 3 blue balls.
Bag B contains 1 red ball and 3 blue balls.
A ball is taken at random from bag A and placed in bag B. A ball is then chosen from bag B. What is the probability that the ball taken from B is red?



16. On a Monday or a Thursday, Mr Gibson paints a 'masterpiece' with a probability of $\frac{1}{5}$. On any other day, the probability of producing a 'masterpiece' is $\frac{1}{100}$. In common with other great painters, Mr Gibson never knows what day it is. Find the probability that on one day chosen at random, he will in fact paint a masterpiece.

17. Two dice, each with four faces marked 1, 2, 3 and 4, are thrown together.
(a) What is the most likely total score on the faces pointing downwards?
(b) What is the probability of obtaining this score on three successive throws of the two dice?

18. In the Venn diagram, $\mathcal{E}$ = {pupils in a class of 15}, G = {girls}, S = {swimmers}, F = {pupils who believe in Father Christmas}.
A pupil is chosen at random. Find the probability that the pupil:
(a) can swim,
(b) is a girl swimmer,
(c) is a boy swimmer who believes in Father Christmas.
Two pupils are chosen at random. Find the probability that:
(d) both are boys,
(e) neither can swim,
(f) both are girl swimmers who believe in Father Christmas.



19. A bag contains 3 red, 4 white and 5 green balls. Three balls are selected without replacement. Find the probability that the three balls chosen are:
(a) all red,
(b) all green,
(c) one of each colour.
If the selection of the three balls was carried out 1100 times, how often would you expect to choose:
(d) three red balls?
(e) one of each colour?

20. There are 1000 components in a box of which 10 are known to be defective. Two components are selected at random. What is the probability that:
(a) both are defective,
(b) neither are defective,
(c) just one is defective?
(Do *not* simplify your answers)

21. There are 10 boys and 15 girls in a class. Two children are chosen at random. What is the probability that:
(a) both are boys,
(b) both are girls,
(c) one is a boy and one is a girl?

22. There are 500 ball bearings in a box of which 100 are known to be undersize. Three ball bearings are selected at random. What is the probability that:
(a) all three are undersize, (b) none are undersize?
Give your answers as decimals correct to three significant figures.

23. There are 9 boys and 15 girls in a class. Three children are chosen at random. What is the probability that:
(a) all three are boys,
(b) all three are girls,
(c) one is a boy and two are girls?
Give your answers as fractions.

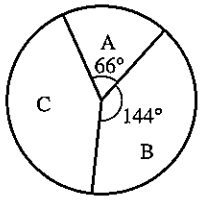
Revision exercise 10A

1. A pie chart is drawn with sectors to represent the following percentages:

20%, 45%, 30%, 5%.

What is the angle of the sector which represents 45%?

2. The pie chart shows the numbers of votes for candidates A, B and C in an election.
What percentage of the votes were cast in favour of candidate C?



3. A pie chart is drawn showing the expenditure of a football club as follows:

Wages	£41 000
Travel	£9 000
Rates	£6 000
Miscellaneous	£4 000

What is the angle of the sector showing the expenditure on travel?

4. The mean of four numbers is 21.
(a) Calculate the sum of the four numbers.
Six other numbers have a mean of 18.
(b) Calculate the mean of the ten numbers.

5. Find:
(a) the mean, (b) the median, (c) the mode,
of the numbers 3, 1, 5, 4, 3, 8, 2, 3, 4, 1.

6.

Marks	3	4	5	6	7	8
Number of pupils	2	3	6	4	3	2

The table shows the number of pupils in a class who scored marks 3 to 8 in a test. Find:
(a) the mean mark,
(b) the modal mark,
(c) the median mark.

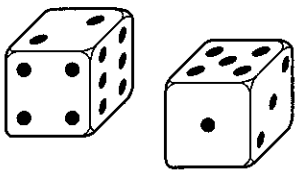
7. The mean height of 10 boys is 1·60 m and the mean height of 15 girls is 1·52 m. Find the mean height of the 25 boys and girls.

8.

Mark	3	4	5
Number of pupils	3	x	4

The table shows the number of pupils who scored marks 3, 4 or 5 in a test. Given that the mean mark is 4·1, find x.

9. When two dice are thrown simultaneously, what is the probability of obtaining the same number on both dice?



10. A bag contains 20 discs of equal size of which 12 are red, x are blue and the rest are white.
(a) If the probability of selecting a blue disc is $\frac{1}{4}$, find x.
(b) A disc is drawn and then replaced. A second disc is drawn. Find the probability that neither disc is red.

11. Three dice are thrown. What is the probability that none of them shows a 1 or a 6?

12. A coin is tossed four times. What is the probability of obtaining at least three 'heads'?

13. A bag contains 8 balls of which 2 are red and 6 are white. A ball is selected and not replaced. A second ball is selected. Find the probability of obtaining:
(a) two red balls,
(b) two white balls,
(c) one ball of each colour.

14. A bag contains x green discs and 5 blue discs. A disc is selected. A second disc is drawn. Find, in terms of x, the probability of selecting:
(a) a green disc on the first draw,
(b) a green disc on the first and second draws, if the first disc is replaced,
(c) a green disc on the first and second draws, if the first disc is *not* replaced.

15. In a group of 20 people, 5 cannot swim. If two people are selected at random, what is the probability that neither of them can swim?

16. (a) What is the probability of winning the toss in five consecutive hockey matches?
(b) What is the probability of winning the toss in all the matches in the FA cup from the first round to the final (i.e. 8 matches)?

17. Mr and Mrs Stringer have three children. What is the probability that:
(a) all the children are boys,
(b) there are more girls than boys?
(Assume that a boy is as likely as a girl.)

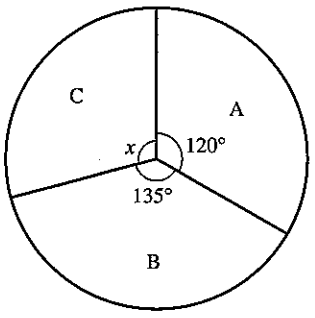
18. The probability that it will be wet today is $\frac{1}{6}$. If it is dry today, the probability that it will be wet tomorrow is $\frac{1}{8}$. What is the probability that both today and tomorrow will be dry?

19. Two dice are thrown. What is the probability that the *product* of the numbers on top is:
(a) 12, (b) 4, (c) 11?

20. The probability of snow on January 1st is $\frac{1}{20}$. What is the probability that snow will fall on the next three January 1st?

Examination exercise 10B

1. In an election, people voted for parties A, B or C. The pie chart shows how the people voted.
(a) Calculate x.
(b) What fraction of the voters voted for party B? (Give your fraction in its lowest terms.)
(c) If 720 people voted for party A, how many voted for party B?
J 96 2



Not to scale

2. The median of $x - 4$, x, $2x$ and $2x + 12$ is 9, where x is a positive integer. Find the value of x. N 98 2

3. The table shows the results of a short test.

mark	0	1	2	3	4	5
number of students	1	3	10	x	6	3

The mode of the marks is 2 and the median mark is 3. Find the possible values of x. N 95 2

4. Answer the whole of this question on a sheet of graph paper.
The table shows the amount of money, $\$x$, spent on books by a group of students.

amount spent ($\$x$)	number of students
$0 < x \leqslant 10$	0
$10 < x \leqslant 20$	4
$20 < x \leqslant 30$	8
$30 < x \leqslant 40$	12
$40 < x \leqslant 50$	11
$50 < x \leqslant 60$	5

(a) Calculate an estimate of the mean amount of money per student spent on books.
(b) Use the information in the table above to find the values of p, q and r in the following cumulative frequency table.

amount spent ($\$x$)	cumulative frequency
$x \leqslant 10$	0
$x \leqslant 20$	4
$x \leqslant 30$	p
$x \leqslant 40$	q
$x \leqslant 50$	r
$x \leqslant 60$	40

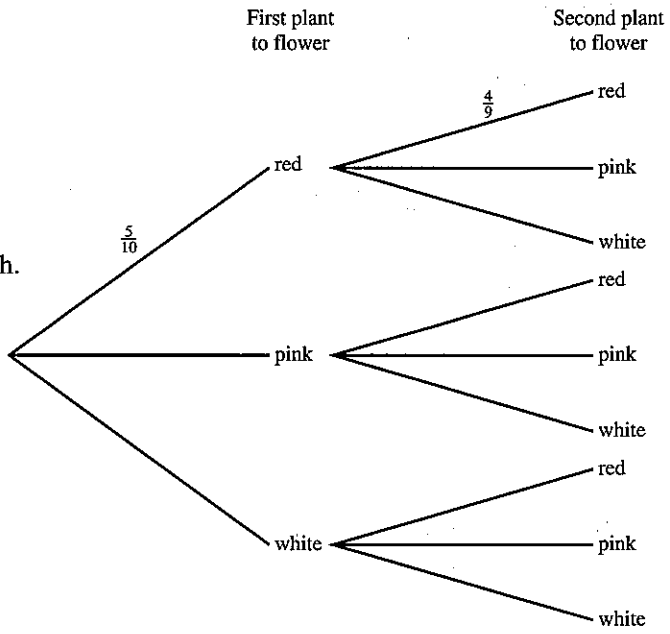
(c) Using a scale of 2 cm to represent 10 units on each axis, draw a cumulative frequency diagram.
(d) Use your diagram:
(i) to estimate the median amount spent,
(ii) to find the upper and lower quartiles, and the inter-quartile range. N 96 4

5. Answer the whole of this question on a sheet of graph paper.
400 apples were weighed. Their masses are given in the table below.

mass (m grams)	frequency
$80 < m \leqslant 100$	50
$100 < m \leqslant 110$	70
$110 < m \leqslant 120$	113
$120 < m \leqslant 130$	92
$130 < m \leqslant 160$	75

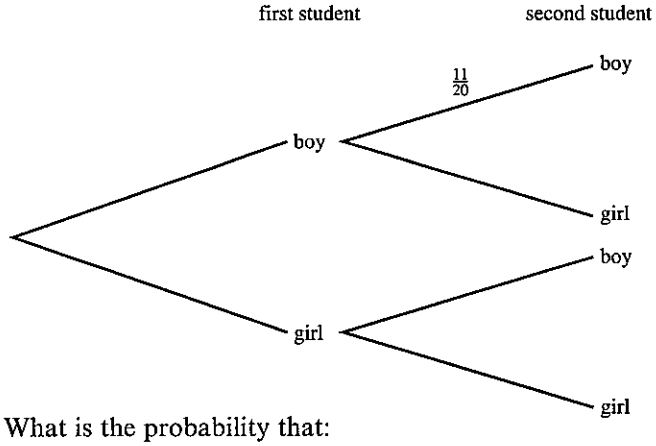
(a) Using a scale of 2 cm to represent 10 g on the horizontal axis, and an *area* scale of 1 cm^2 to represent 5 apples, draw a histogram to display this data.
(b) Calculate an estimate of the mean mass of the apples.
(c) A supermarket will only buy apples which have a mass greater than 110 g. What percentage of the apples does the supermarket buy? N 98 4

6. Give each of your answers to this question as a fraction.
Peter has 10 geranium plants. He knows that five will flower red, three pink and two white.
(a) What is the probability that the first plant to flower is pink?
(b) Copy the tree diagram. Write the correct probability on each branch.
(c) What is the probability that, of the first two plants to flower:
(i) both are red,
(ii) one is red and the other is pink,
(iii) at least one is pink?
(d) What is the probability that the first three plants to flower are all white? N 98 4



7. Give your answers to this question as fractions in their lowest terms. There are 21 students in a class. 12 are boys and 9 are girls. The teacher chooses two students at random.

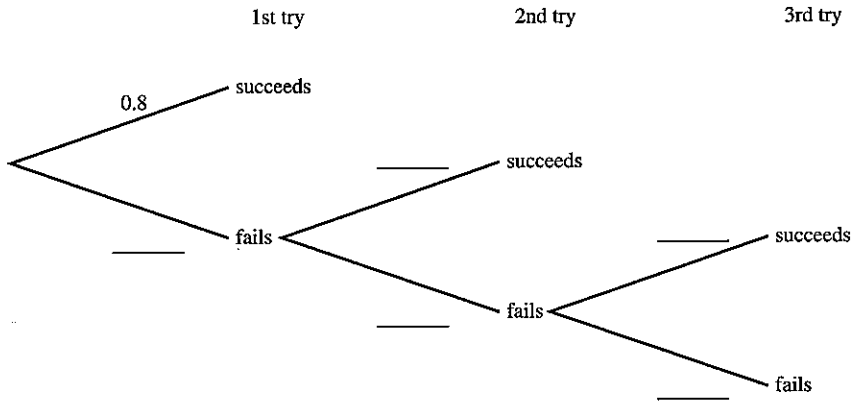
(a) If the first student chosen is a boy, explain why the probability that the second student chosen is also a boy is $\frac{11}{20}$.
(b) Copy the tree diagram below. Write the correct probability on each branch.



(c) What is the probability that:
(i) both students are boys,
(ii) both students are girls,
(iii) one is a boy and one is a girl?
(d) The teacher chooses a third student at random. What is the probability that;
(i) all three students are boys,
(ii) at least one of the three students is a girl? J 96 4

8. Mamoud tries to repair a broken toy. Each time he tries the probability that he succeeds is 0·8. Each time he fails he tries again.

(a) Copy and complete the tree diagram below.



(b) Find the probability that, to succeed, it takes:
(i) exactly two tries,
(ii) one, two or three tries,
(iii) exactly five tries.

(c) Write down a formula for the probability that he has not succeeded after n tries. J 89 4

9. Answer the whole of this question on a sheet of graph paper.
120 passengers on an aircraft had their baggage weighed. The results are shown in the table.

Mass of baggage (M kg)	Number of passengers
$0 < M \leqslant 10$	12
$10 < M \leqslant 15$	32
$15 < M \leqslant 20$	28
$20 < M \leqslant 25$	24
$25 < M \leqslant 40$	24

(a) (i) Write down the modal class.
(ii) Calculate an estimate of the mean mass of baggage for the 120 passengers. Show all your working.
(iii) Sophia draws a pie chart to show the data.
What angle should she have in the $0 < M \leqslant 10$ sector?

(b) Using a scale of 2 cm to represent 5 kg, draw a horizontal axis for $0 < M \leqslant 40$.
Using an area scale of 1 cm^2 to represent 1 passenger, draw a histogram for this data. N 03 4