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Oxford University Press is a worldwide publisher of educational textbooks and we have made full use of our international reputation to bring you the most authoritative text for the Extended IGCSE syllabus.

About this book

This book is designed to provide the best preparation for your IGCSE examination. It is written by a very popular and successful author, and is fully endorsed for the CIE syllabus so you can be sure it covers everything you need to know.

Finding your way around

To get the most out of this book when studying or revising, use the:

- **Edge marks** to help you find the unit you want quickly.
- **Contents list** to help you find the appropriate units.
- **Index** to find key words so you can turn to any concept straight away.

Exercises and exam questions

There are literally thousands of questions in this book, providing ample opportunities to practise the skills and techniques required in the exam.

- **Worked examples and comprehensive exercises** are main features of the book. The examples show you the important skills and techniques required. The exercises are carefully graded, starting from the basics and going up to exam standard, allowing you to practise the skills and techniques.
- **Revision exercises** at the end of each unit allow you to bring together all your knowledge on a particular topic and encourage regular revision.
- **Examination exercises** at the end of each unit consist of questions from past IGCSE papers. They are coded so you can tell immediately which paper they are taken from: [J 95 2] means the question is from June 95 Paper 2; [N 98 4] is from November 98 Paper 4.
- **Specimen exam papers** at the end of the book are written by the Principal Examiner. There are two papers, corresponding to the papers you will take at the end of your course: Paper 2 and Paper 4. They give you the opportunity to practise for the real thing.
- **Revision section:** Unit 12 contains multiple choice questions to provide an extra opportunity to revise, making sure you are completely ready for your exam.
- **Answers to numerical problems** are at the end of the book so you can check your progress.

Investigations

Unit 11 provides plenty of ideas to help you gain the special skills required for the Investigation paper. Remember that you can only gain by taking this optional paper – you cannot lose marks – so it is worth developing these skills.

Links to curriculum content

At the start of each unit you will find a list of objectives that are covered in the unit. These objectives are drawn from the Core and Supplement as described in the IGCSE Mathematics syllabus.

- 17** Apply rate of change to distance–time and speed–time graphs
- 18** Construct tables of values and draw graphs for functions of the form ax^n ; estimate gradients of curves by drawing tangents
- 19** Interpret and obtain the equation of a straight-line graph in the form $y = mx + c$; calculate the gradient of a straight line from the coordinates of two points on it

The number attached to each objective is the topic number as given in the syllabus provided by the CIE examination board. It is for ease of reference and to help you to plan your study.

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